# Analytical Compensation of Inter-reflection for Pattern Projection

Yasuhiro Mukaigawa
Osaka University
8-1 Mihogaoka, Ibaraki,
Osaka, Japan
mukaigaw@ieee.org

Takayuki Kakinuma
University of Tsukuba
1-1-1 Tennodai, Tsukuba,
Ibaraki, Japan

Yuichi Ohta
University of Tsukuba
1-1-1 Tennodai, Tsukuba,
Ibaraki, Japan
ohta@acm.org

## ABSTRACT

If a pattern is projected onto a concave screen, the desired view cannot be correctly observed due to the influence of inter-reflections. This paper proposes a simple but effective technique for photometric compensation in consideration of inter-reflections. The compensation is accomplished by canceling inter-reflections estimated by the radiosity method. The significant advantage of our method is that iterative calculations are not necessary because it analytically solves the inverse problem of inter-reflections.

**Categories and Subject Descriptors**
I.4.8[**Scene Analysis**]: Photometry;
**General Terms:** Algorithms
**Keywords:** projector, inter-reflection

## 1. INTRODUCTION

As projectors have become smaller and cheaper, they have been used for various purposes and new applications beyond presentation or display to a large screen have been researched. In general, it is premised that patterns are projected to the limited surface such as a white planar or a dome screen, but if we can use a general surface such as corners of a room or 3D objects as a screen, then the capability of the projector can be extended. For example, Raskar et al.[1] proposed a system that can display information onto an arbitrary surface in a room. To use all surrounding surfaces as a screen, we have to solve two major problems.

The first problem is that the screen is not white. If the surface has a uniform and light color, color compensation compatible with the surface color has already been commercialized. However, where the surface has texture, correct views cannot be seen because the projected pattern is mixed with the texture. Nayar et al.[2] proposed a technique for pattern compensation according to the texture. Fujii et al.[3] proposed a real-time compensation method. Wang et al.[4] proposed a method which minimizes perceptual artifacts by taking the human vision system into account. By using these accomplishments, the compensated pattern can make the light texture of the screen invisible.

The second problem is that the screen is not flat. The projected light is attenuated, if it is not perpendicular to the surface of the screen and spatial brightness varies due to the differences in the incident angle of the light. In addition, the influence of inter-reflection is also pronounced, as the projected light on certain surfaces illuminates other surfaces if the shape of the screen is concave. If these problems are solved, the projector can be used as a new type of display device because the geometric restriction of the screen is greatly reduced. For example, to simulate virtual reflectance properties and virtual illumination effects in the real world, many mixed reality systems have been proposed [5][6][7][8] in which 3D objects were used as a screen. However, these systems did not take inter-reflection into account at all. Recently, Bimber et al. [9] proposed a method to compensate inter-reflections based on the radiosity method [10]. However, their method requires iterative calculations because the inter-reflection problem is not analytically solved.

In this paper, we show a fast calculation algorithm for compensating inter-reflections without iterative calculations. In the computer vision field, Fournier et al.[11] proposed a photometric analysis method which takes inter-reflections into account and our approach is similar to this idea. By analytically solving the inverse problem of inter-reflections, our method does not require any iterative calculation. Moreover, our method can treat not only 2-bounce reflections, but also the multiple-bounce ones.


*VRST'06*, November 1–3, 2006, Limassol, Cyprus.


## 2. COMPENSATION ALGORITHM

### 2.1 System Configuration

Let's consider the situation that a pattern is projected onto a concave surface. In this case, the reflected light on the screen surface illuminates other surfaces again. In this research, we focus on how to reduce the influence of inter-reflections and simplify the task setting. Figure 1 shows the system configuration. We use a set of a projector and a range finder, and assume that the non-planar screen has a white Lambertian surface. We also assume that the user's position is fixed and known.

### 2.2 Compensation of Direct Illumination

First of all, we assume the simplest case where inter-reflection does not occur. Now, we consider the ideal situation that the projection value (video signal to the projec-

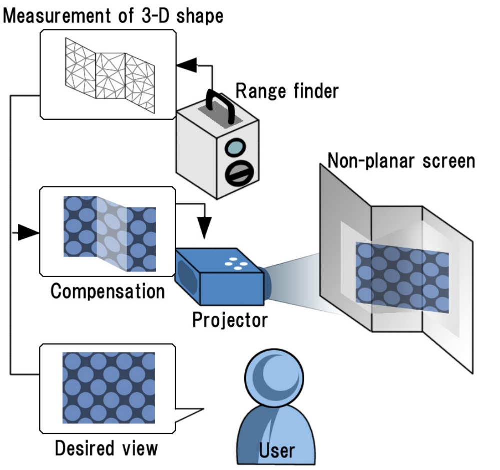



**Figure 1: System configuration.**

tor) corresponding a white patch $i$ is $p_i$, and that the patch is illuminated from the direction of the surface normal by illuminance $L_i$ and desired radiance $c_i$ is obtained. In general, the projection value and the illuminance on the screen have a nonlinear relationship that depends on the individual equipment. Therefore, we directly measure the illuminance of the projector light using an illuminometer. This relationship can be formulated using function $f$ as follows,

$$L_i = f(p_i), \tag{1}$$

If the reflectance is $r$, the radiance is expressed as follows,

$$c_i = rL_i. \tag{2}$$

Next, we assume that the patch $i$ leans $\theta_i$ from the direction of the projection. Then, the radiance $c'_i$ attenuates according to the incident angle as follows,

$$c'_i = rL_i \cos\theta_i. \tag{3}$$

That is, we can calculate the influence of the attenuation in advance, if $\theta_i$ is known. The illuminance $L_i$ is compensated to $\hat{L}_i$ as follows,

$$\hat{L}_i = \frac{c_i}{c'_i} L_i = \frac{L_i}{\cos\theta_i}. \tag{4}$$

We can obtain the desired radiance $c_i$ by projecting this compensated illuminance $\hat{L}_i$ to the patch. Since the function $f$ is a monotonically increasing function, the inverse function $f^{-1}$ can be easily defined. Hence, the actual projection value $\hat{p}_i$ to illuminate $\hat{L}_i$ can be calculated by

$$\begin{aligned} \hat{p}_i &= f^{-1}(\hat{L}_i) && (5)\\ &= f^{-1}\left(\frac{L_i}{\cos\theta_i}\right) && (6)\\ &= f^{-1}\left(\frac{f(p_i)}{\cos\theta_i}\right). && (7) \end{aligned}$$

## 2.3 Inter-reflection and Radiosity

Next, we assume more general situations in which inter-reflection occur. The influence of the inter-reflection can be directly measured by a laser scan when the geometry of the screen is unknown[12]. However, we can calculate the influence of the inter-reflection based on the radiosity [10] because the 3D shape of the screen has already been measured for the geometric conversion of the projection pattern.

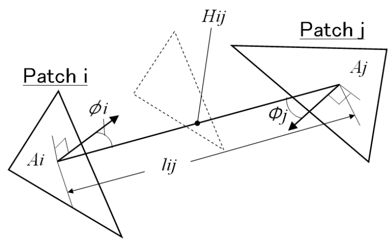



**Figure 2: Form Factor between two patches.**

Radiosity is a method for calculating the propagation of energy in the entire scene based on the form factor. The form factor $F_{ij}$ denotes the propagation ratio of energy that patch $i$ receives from patch $j$ as shown in Figure 2, and it is calculated based on the distance $l_{ij}$ between $i$ and $j$, the angle $\phi_i$, $\phi_j$ between each surface normal and vector from $i$ to $j$, the area of $i$ and $j$, and the coefficient $H_{ij}$ which denotes the existence of an obstacle between $i$ and $j$ as the following equation,

$$F_{ij} = \frac{1}{\pi A_i} \int_{A_i} \int_{A_j} H_{ij} \frac{\cos\phi_i \cos\phi_j}{l_{ij}^2} dA_j dA_i \tag{8}$$

Since the form factor is independent of the illuminance and the reflectance, it is calculated only by geometry. The radiance $c'_i$ which takes inter-reflection into account based on the form factor is calculated by

$$c'_i = rL_i \cos\theta_i + r\sum_j F_{ji} c'_j. \tag{9}$$

## 2.4 Compensation for Inter-reflection

In this section, we propose an analytical method to compensate inter-reflection based on the radiosity. The most significant merit is that the compensation is very fast because it does not require any iterative calculation.

Equation (9) indicates the situation where the wrong radiance $c'_i$ is obtained by the unsuitable illuminance $L_i$. Now we want to make the ideal situation where the correct radiance $c_i$ is obtained by the compensated illuminance $\hat{L}_i$ as follows,

$$c_i = r\hat{L}_i \cos\theta_i + r\sum_j F_{ji} c_j. \tag{10}$$

It is important that Eq.(2) remains valid because it describes just the relationship between illuminance and radiance. The relationship is independent of the existence of inter-reflections. Therefore, the compensated illuminance $\hat{L}_i$ is calculated by

$$\begin{aligned} \hat{L}_i &= \frac{c_i - r\sum F_{ji} c_j}{r\cos\theta_i} && (11)\\ &= \frac{rL_i - r\sum F_{ji} rL_j}{r\cos\theta_i} && (12)\\ &= \frac{L_i - r\sum F_{ji} L_j}{\cos\theta_i} && (13)\\ &= \frac{f(p_i) - r\sum F_{ji} f(p_j)}{\cos\theta_i}. && (14) \end{aligned}$$

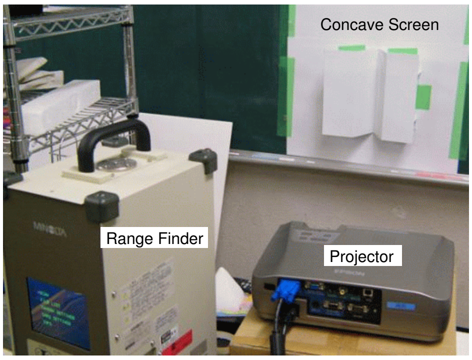



**Figure 3: Overview of the system.**

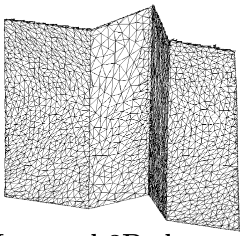


**Figure 4: Measured 3D shape of the screen.**

The compensated projection value to the patch $i$ is calculated by

$$\hat{p_i} \quad = \quad f^{-1}\left(\frac{f(p_i) - r\sum F_{ji}f(p_j)}{\cos\theta_i}\right). \tag{15}$$

In this algorithm, the compensated pattern is directly calculated from Eq.(15) without any iterative calculation. Moreover Eq.(15) can compensate for multiple-bounce reflections.

# 3. EXPERIMENTAL RESULTS

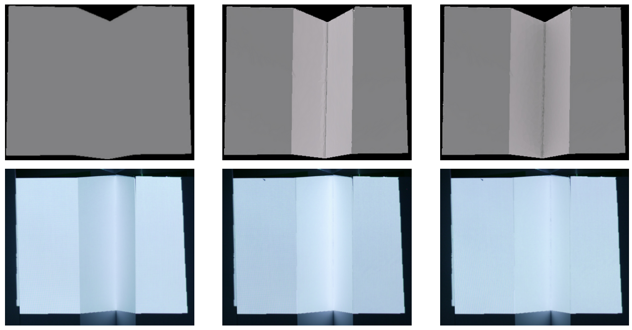
## 3.1 System Configuration

We used a projector (EPSON EMP-74) and a range finder (KONICA MINOLTA VIVID910) as shown in Figure 3. The screen has a convex part whose angle is about 60°. The 3D shape of the screen was modeled as a set of 4200 triangular patches as shown in Figure 4. Although we used a range finder to keep accuracy, it is possible to substitute the range finder with the projector and camera set [8].

The geometric calibration of the range finder and the projector, measurement of the 3D surface of the scene, and calculation of the form factor are done in advance as a preprocess. Hence, the photometric compensation and the projection can be done in real time.

## 3.2 Gray Pattern

First, we evaluated the effectiveness of the proposed algorithm by compensating uniform gray patterns. Figure 5 shows results of the comparative experiments. The upper row shows the projected patterns and the lower row shows the appearance of the screen, respectively. (a) is the result without any compensation and the projected light is attenuated at the concave part. (b) is the result with the compensation of only direct illumination. Although the attenuation at the concave part is compensated, the center area incorrectly becomes brighter due to the inter-reflection. (c) is the result of our proposed method which compensates both the direct illumination and the inter-reflection and the appearance of the screen is almost uniform. For the quantitative evaluation variances of the pixel intensities in the concave area are calculated as shown in Table 1. Clearly our proposed method has the smallest variance.



(a) (b) (c)

**Figure 5: Compensation for monochrome pattern. (a) no compensation, (b) compensation of direct illumination, (c) proposed method**

**Table 1: Variance at concave part**

| | |
|---|---|
| **No compensation** | 88.1 |
| **Compensation of direct illumination** | 60.5 |
| **Proposed method** | 21.7 |

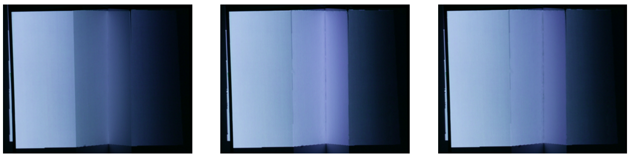
Next, we projected a gradation pattern to analyze the spatial change of the appearance. Figure 6 shows the comparison of the appearance of the screen. The pixel intensity is scanned horizontally, and the intensity changes are shown in Figure 7. This graph clearly indicates the features of the three appearances. We can see that our method can project the smoothest gradation pattern.

## 3.3 Color Pattern

The proposed algorithm is designed to compensate only illuminance. However, color patterns can be also compensated by applying the algorithm to each R,G and B channel independently, if we can assume that the spectral distribution corresponding to R, G, and B do not overlap.

Figure 8(a) is an original color pattern. The left half is divided into three stripes of R, G and B, and the right half is uniform gray so that we can easily confirm the effectiveness. In the compensated pattern (b), the opposite side to the green changes the color from gray to cyan because this gray part is predicted to be affected by indirect green illumination. In fact, the right half of the actual appearance is a uniform gray as shown in (c).

For a quantitative evaluation, we analyzed the hue of the right part of the screen. The transition of the hue is checked along the vertical line between the ▲ and ▼ marks in Fig.8(c) where the inter-reflection is strongly observed. Figure 9 shows the transition of the hue. Since the hue of the proposed method is almost uniform compared with the other two results, we can confirm that the influence from inter-reflection is drastically reduced.



(a) (b) (c)

**Figure 6: Compensation for gradation pattern. (a) no compensation, (b) compensation of direct illumination, (c) proposed method**

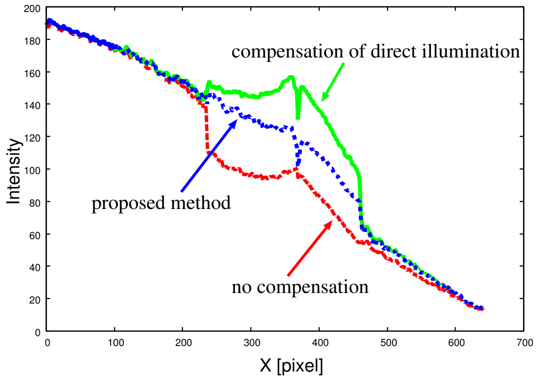
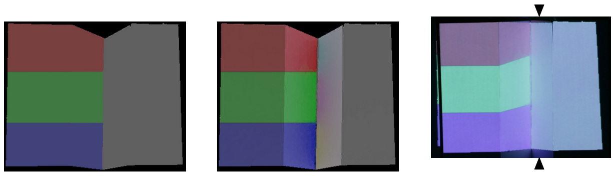



**Figure 7: Transition of intensity along horizontal line.**



(a) Original (b) Compensated (c) Actual view

**Figure 8: Compensation for color pattern.**

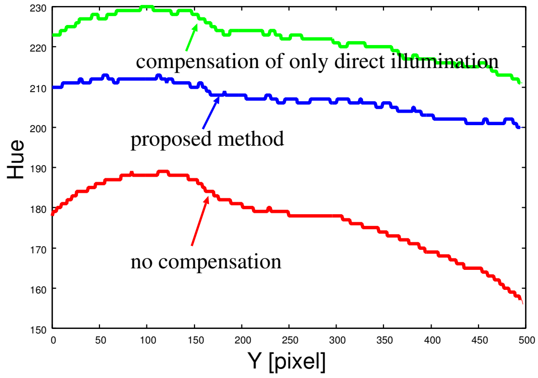



**Figure 9: Transition of hue along the concave edge.**

In this experiment, compensation could be undertaken within 0.12s for each pattern using a Pentium4 2.53GHz. This speed is fast enough for many purposes such as a presentation. Moreover, if the number of patches of the 3D screen shape can be reduced, it will be possible to compensate for inter-reflection in video in real time.

## 4. CONCLUSION

In this paper, we proposed a new method for compensating the influence of the inter-reflection for the pattern projection to a non-planar screen. Since our method is based on the radiosity which is calculated by the geometry of the screen, the projected pattern is compensated without any iterative computation. Since the compensated time is about 0.12s, our method can be applied for real time applications. Moreover our method can reduce the limitation that projected screen should be a flat plane.

Our future work is to develop a faster compensation system for projecting movie onto non-planar screens. Measuring colors of the screen using a chromameter is also necessary to confirm the effectiveness of the proposed algorithm.

## 5. REFERENCES

[1] R. Raskar, G. Welch, M. Cutts, A. Lake, L. Stesin, and H. Fuchs, "The Office of the Future : A Unified Approach to Image-Based Modeling and Spatially Immersive Displays", Proc. ACM SIGGRAPH, pp.179-188, 1998.

[2] S. K. Nayar, H. Peri, M. D. Grossberg, and P. N. Belhumeur, "A Projection System with Radiometric Compensation for Screen Imperfections" Proc. International Workshop on Projector-Camera Systems (PROCAMS2003), 2003.

[3] K. Fujii, M. Grossberg, and S. Nayar, "A Projector-Camera System with Real-Time Photometric Adaptation for Dynamic Environments", Proc. Computer Vision and Pattern Recognition (CVPR2005), Vol.1, pp. 814-821, 2005

[4] D. Wang, I. Sato, T. Okabe, and Y. Sato, "Radiometric Compensation in a Projector-Camera System Based on the Properties of Human Vision System", Proc. International Workshop on Projector-Camera Systems (PROCAMS2005), 2005.

[5] R. Raskar, G. Welch, and W. C. Chen, "Table-Top Spatially-Augmented Reality: Bringing Physical Models to Life with Projected Imagery", Proc. International Workshop on Augmented Reality'99 (IWAR'99), pp.64-71, 1999.

[6] R. Raskar, G. Welch, K. Low, and D. Bandyopadhyay, "Shader Lamps: Animating Real Objects With Image-Based Illumination", Proc. Eurographics Rendering Workshop (ERGW2001), pp.89-102, 2001.

[7] Y. Mukaigawa, M. Nishiyama, and T. Shakunaga, "Virtual Photometric Environment using Projector", Proc. International Conference on Virtual Systems and Multimedia (VSMM2004), pp.544-553, 2004.

[8] Y. Mukaigawa, H. Nagai, and Y. Ohta, "SpaceRelighter -Recording and Reproducing Illumination in a Real Scene", Proc. International Conference on Virtual Systems and Multimedia (VSMM2004), pp.109-118, 2004.

[9] O. Bimber, A. Grundhöfer, T. Zeidler, D. Danch, and P. Kapakos, "Compensating Indirect Scattering for Immersive and Semi-Immersive Projection Displays", Proc. IEEE Virtual Reality Conference (VR2006), pp.151-158, 2006.

[10] F. Sillion, and C. Puech, "Radiosity and global illumination", Morgan Kaufma nn, San Francisco, CA, 1994.

[11] A. Fournier, A. S. Gunawan, and C. Romanzin, "Common illumination between real and computer generated scenes", Proc. Graphics Interface '93, pp. 254-262, 1993.

[12] S. M. Seitz, Y. Matsushita, and K. N. Kutulakos, "A Theory of Inverse Light Transport", Proc. International Conference on Computer Vision (ICCV2005), Vol.2, pp.1440-1447, 2005.