Acquiring Non-parametric Scattering Phase Function from a Single Image

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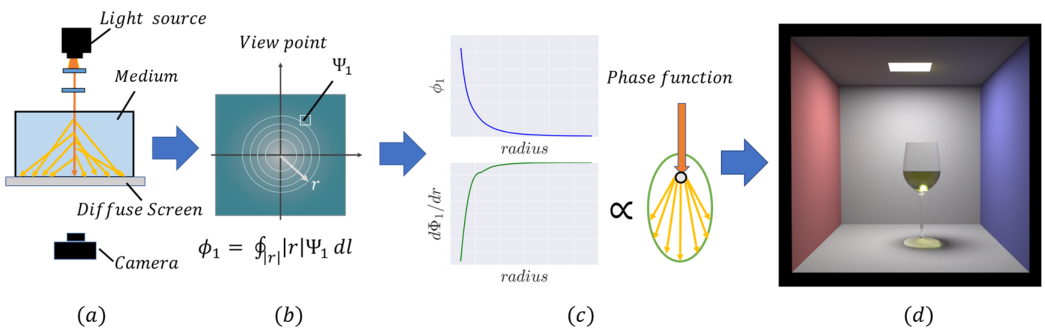

Figure 1: Workflow for measuring the non-parametric phase function. (a) The scattering distribution is obtained from a single camera image. (b) The integral Φ_1 is calculated from the total flux of the scattered light Ψ_1 over a circle of radius r on the acquired image. (c) According to our theory, partial derivative of Φ_1 with respect to r is proportional to the phase function. (d) Rendering image of the media using the measured phase function.

ABSTRACT

Acquiring accurate scattering properties is critical for rendering translucent materials such as participating media. In particular, the phase function, which determines the distribution of scattering directions, plays a significant role in the appearance of the material. While there are many techniques to acquire BRDF, there are only a few methods for the non-parametric phase function. We propose a distinctive scattering theory that approximates the effect of single scattering to acquire the non-parametric phase function from a single image. Furthermore, in various experiments, we measured the phase functions from several real diluted media and rendered images of these materials to evaluate the effectiveness of our theory.

CCS CONCEPTS

• **Computing methodologies** → *Appearance and texture representations*;

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1 INTRODUCTION

Participating media exists everywhere in our surroundings. To reproduce a realistic computer graphics of a medium, the scattering phenomena occurring inside the material needs to be described accurately. The rendering of the scattering material depends on several of its physical characteristics. The phase function is one such characteristic and determines the distribution of scattered light direction. Therefore, measuring the phase function of a real object is important in portraying its appearance realistically.

Similar to the phase function, the bidirectional reflectance distribution function (BRDF) also describes the distribution of reflected light. Although, recent studies on measuring BRDF adopt a non-parametric representation, phase-function measurements usually adopt parametric models such as the Henyey-Greenstein (HG)

Table 1: Symbols used in equations.

$\hat{\boldsymbol{r}}, \boldsymbol{r}, z$	position ($\hat{\boldsymbol{r}} = \boldsymbol{r} + z\hat{\boldsymbol{z}}$)
$\hat{\boldsymbol{s}}, \boldsymbol{s}$	direction ($\hat{\boldsymbol{s}} \simeq \boldsymbol{s} + \hat{\boldsymbol{z}}$)
σ_t	extinction coefficient
σ_s	scattering coefficient
$p(\boldsymbol{s})$	phase function

phase function[Henyey and Greenstein 1941], in many cases. The parametric model, in an approximate manner, can easily reproduce phenomena from a small number of parameters. However, it sacrifices some physics plausibility and there are limitations in its capacity to generate correctly a realistic appearance of actual media. Nevertheless, Hawkins *et al.*[2005] proposed a method to measure the phase function directly. However, it requires extensive measurements using special equipment to acquire the variety of angular data.

We hereby propose a method for acquiring the non-parametric phase function of a real media from a single image. Our method is based on a distinctive formulation of the relationship between the single scattered field and the phase function. To validate our method, we prepare a projector-camera optical system that enables the phase function to be measured from a single image captured with simple equipment.

In summary, the contributions of this paper are as follows:

- A formulation of single scattering fields is proposed.
- A simple method is proposed to measure the non-parametric phase function from just a single image.
- Experiments using simple equipment have been performed demonstrating the technique.

2 RELATED WORK

In the field of graphics, acquiring scattering properties from real media has been studied. Jensen *et al.*[2001] obtained scattering properties using the diffusion approximation. The method assumes that multiple scattering is occurring with the optically dense medium, so that certain characteristics of the medium from scattering properties can be estimated but not the phase function.

Narasimhan *et al.*[2006] measured scattering properties using the single scattering approximation. Their method estimates scattering properties using the minimizing error function about image formulation model of single scattering and observation. This method represents the phase function as a HG phase function. We believe the HG function is insufficient in reproducing the properties of actual media[Gkioulekas et al. 2013a].

Gkioulekas *et al.*[2013b] measured scattering properties by solving the problem of appearance matching. They proposed to represent the phase function as a convex combination of a tent function and a weight factor. This function is more flexible than the HG phase function. To ensure the estimation is accurate, it requires many observations using a special optical system that requires a complex calibration process.

Whereas the above methods adopted the parametric model for representing the phase function, other methods adopt a non-parametric approach. Hawkins *et al.*[2005] measured scattering properties by developing a laser-scanning system. Because there is no model for the phase function, their system directly measures the phase function from a wide range of directions. Although they built special equipment with a conical mirror, scattering from some directions were unable to be measured, specifically scattering angles ≈ 0 and π of the phase function. In these instances, an optical element used in the measurement blocks the path of the ray. Moreover, it is not easy to split the measurement into scattering and direct components.

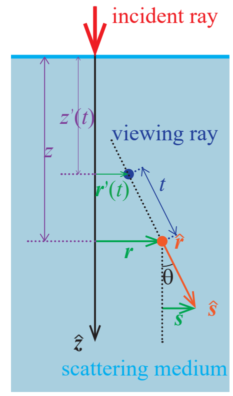


Figure 2: Coordinate system and experimental arrangement.

3 FORMULATION OF SCATTERED FIELDS

In this section, we derive a direct and simple relationship between single scattering fields and the phase function, which then permits a one-shot measurement of phase functions. The main symbols used in the equations are listed in Table 1.

Consider the situation shown in Fig. 2; a perpendicular incident ray scatters in an uniform participating medium. For a simpler formulation, let us assume:

- the medium is optically thin, and single scattering is dominant,
- the medium produces forward scattering, and
- the phase function of the medium is axial-symmetrical depending only on the angle between the directions of incident and scattered light.

The first assumption allows the single-scattering approximation to be imposed. This can be satisfied by controlling the density of the medium during measurements. From the second, the angle θ between the scattered light and the ray axis $\hat{\boldsymbol{z}}$ can be regarded to be small, and the cosine can be approximated as

$$\hat{\boldsymbol{s}} \cdot \hat{\boldsymbol{z}} = \cos\theta \approx 1.$$

As reported in previous works [Gkioulekas et al. 2013b; Narasimhan et al. 2006], most participating media were found to be dominantly forward-scattering, and thus comply with this assumption. The third assumption relates to the symmetry of the scattering and is commonly made in both measurements and image generation. There is though no significant lost in generality.

The direct light I_0 measured at position $\boldsymbol{r}$, depth z, in the direction $\boldsymbol{s}$ is given by

$$I_0(z, \boldsymbol{r}, \boldsymbol{s}) = \delta(\boldsymbol{r})\delta(\boldsymbol{s})e^{-\sigma_t z}, \quad (1)$$

where $\delta()$ denotes the Dirac delta function. Note that $I_0(z, \boldsymbol{r}, \boldsymbol{s}) \neq 0$ where $\boldsymbol{r} = \boldsymbol{0}$ and $\boldsymbol{s} = \boldsymbol{0}$, which is along the incident light. The single scattering field I_1 can then be calculated by a line integral along a viewing ray denoted by

$$z'(t) = z - t(\hat{z} \cdot \hat{s}) \simeq z - t \quad (2)$$

$$\boldsymbol{r}'(t) = \boldsymbol{r} - t\boldsymbol{s}. \quad (3)$$

Therefore,

$$\begin{aligned} I_1(z, \boldsymbol{r}, \boldsymbol{s}) &= \frac{\sigma_s}{4\pi} \int_0^z e^{-\sigma_t t} \int_\Omega I_0(z'(t), \boldsymbol{r}'(t), \boldsymbol{s}')p(|\boldsymbol{s} - \boldsymbol{s}'|)d\boldsymbol{s}'dt \\ &= \frac{\sigma_s}{4\pi} e^{-\sigma_t z} \int_0^z \delta(\boldsymbol{r} - t\boldsymbol{s})p(|\boldsymbol{s}|)dt, \quad (4) \end{aligned}$$

where Ω represents a unit sphere, and the phase function $p(|\boldsymbol{s} - \boldsymbol{s}'|)$ is normalized to 4π.

$$\int_\Omega p(|\boldsymbol{s} - \boldsymbol{s}'|)d\boldsymbol{s} = 4\pi \quad (5)$$

The total flux Ψ_1 is then obtained by integrating Eq. 4 over $\boldsymbol{s}$:

$$\begin{aligned} \Psi_1(z, \boldsymbol{r}) &= \int_\Omega I_1(z, \boldsymbol{r}, \boldsymbol{s}) \cos\theta d\boldsymbol{s}, \\ &= \frac{\sigma_s}{4\pi} e^{-\sigma_t z} \int_0^z \int_\Omega \delta(\boldsymbol{r} - t\boldsymbol{s})p(\boldsymbol{s})d\boldsymbol{s}dt, \quad (6) \\ &= \frac{\sigma_s}{4\pi} \frac{1}{|\boldsymbol{r}|} e^{-\sigma_t z} \int_{|\boldsymbol{r}|/z}^\infty p(u')du', \quad (7) \end{aligned}$$

where $u = 1/t$ and $u' = |\boldsymbol{r}|u$. Note that the flux Ψ_1 depends on the distance from the axis, $|\boldsymbol{r}|$, and is constant on a circle $|\boldsymbol{r}| =$ constant. To remove the apparent singularity from the factor $1/|\boldsymbol{r}|$, we integrate Ψ_1 over the circle, which yields

$$\begin{aligned} \Phi_1(z, |\boldsymbol{r}|) &= \oint_{|\boldsymbol{r}|} |\boldsymbol{r}|\Psi_1(z, \boldsymbol{r})d\boldsymbol{r} \\ &= \frac{\sigma_s}{2} e^{-\sigma_t z} \int_{|\boldsymbol{r}|/z}^\infty p(u')du. \quad (8) \end{aligned}$$

This states an important fact that Φ_1 is *proportional to an integral of the phase function*. By taking the derivative with respect to $|\boldsymbol{r}|$ on both sides, we obtain

$$\frac{\partial \Phi_1}{\partial \boldsymbol{r}} = -\frac{\sigma_s}{2z} e^{-\sigma_t z} p\left(\frac{|\boldsymbol{r}|}{z}\right). \quad (9)$$

Note that $|\boldsymbol{r}|/z$ represents the scattering angle.

The flux integral expressed by Eq. 7 can be optically realized using a diffusing screen. Therefore, Eq. 9 suggests the following simple one-shot measurement of the phase function,

(1) set a diffusing screen on the boundary of the medium,
(2) set a camera on the incident ray axis and image the scattering field,
(3) calculate the circular integral (Eq. 8) from the image, and
(4) calculate its derivatives.

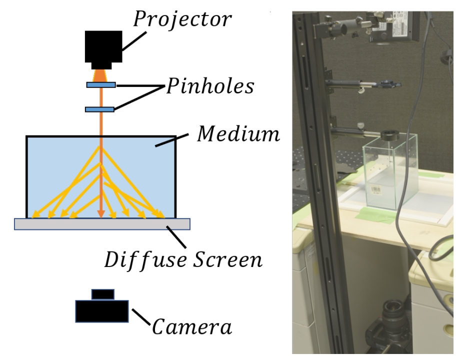


Figure 3: Experimental setup.

4 EXPERIMENT

4.1 Experimental setup

In this section, we describe the experimental setup that we erected following the requirements of the formulation proposed for measuring the non-parametric phase function of real media.

4.2 Setup

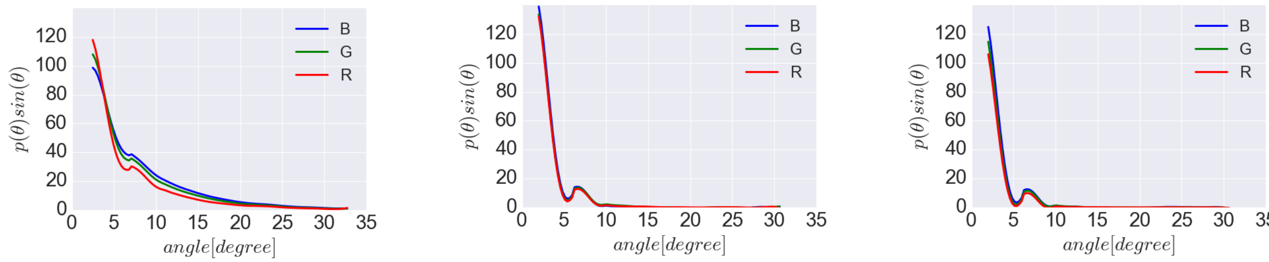
A schematic illustration and a photograph of the experimental setup is presented in Fig. 3. In the formulation, the incident ray is considered to be a narrow beam. To ensure the beam is narrow, we used an LED projector (Optoma ML750) and two pinholes of the same size (0.4 mm). We setup the projector to direct a beam of white light, through both pinholes and mount them over a water tank ensuring that the beam is incident perpendicularly to the surface of the participating media. A diffuse screen is set under the tank to integrate the flux of scattered light. A DSLR camera (NIKON D5300) is placed under the screen to image the scattered light. In total, our method requires only a projector, a camera, a diffuse screen, a tank, two pinholes, and holders for each of them. Preparing the setup is very easy compared with setups of previous studies measuring the non-parametric scattering phase function.

We confirmed that the setup functioned correctly by testing several media (milk, apple juice, and chardonnay wine) representing different types of characteristics (optically thick and optically thin). To comply with the assumption that single scattering is dominant, the media was diluted in water. The diluted media was poured into the tank to a depth of 50 mm.

4.3 Experimental Results

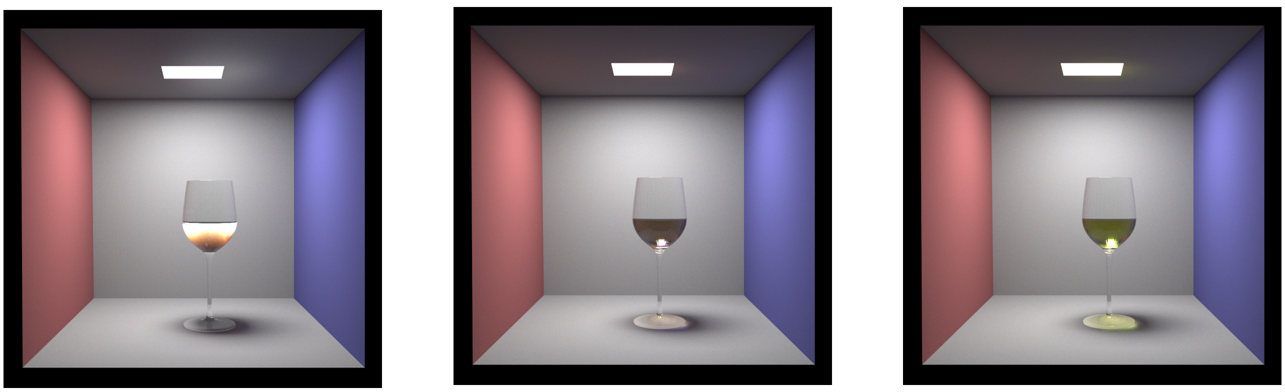
The measured phase function is presented in Fig. 4. The RGB color channels of the captured image are independently used for calculating the phase function. The measured phase functions are different for each medium; differences arising from color for one medium are relatively small but still can not be ignored.

There are a few limitations in our method. The major limitation is that, as found with other methods, it is difficult to measure the phase function of the scattering angle ≈ 0. In real settings, direct light is much stronger than scattered light and causes bleeding of light on the screen. Thus, the scattered light near the center part is biased and affects the phase function as well.



Milk Apple juice Chardonnay wine

Figure 4: Measured phase functions.



Milk Apple juice Chardonnay wine

Figure 5: Rendered images.

4.4 Rendering image

Figure 5 presents the rendered image of measured medium in the glass. We use the path tracing algorithm for rendering. Image size is 1024 × 1024 and using 20,000 photon samples. These images are rendered by using measured non-parametric phase function as tabulated data. The scattering and extinction coefficients are manually determined. Furthermore, since we can not measure whole direction, we complemented the missing value of the phase function from the measured data for the rendering purpose.

5 CONCLUSION

We have proposed a method that acquires the non-parametric phase function from a single image. Our setup for the measurement requires neither special equipment nor complicated calibrations. Also, we have presented rendered images of several different media using the measured phase functions.

Our proposed model still has a few limitations. Because we ignored the effect of higher-order scattering, we diluted the media to decrease the scattering effect. We also assumed that the scattering distribution is concentrated in the forward direction so that the phase function of the media corresponding to back scattering need not be measured.

While we can now measure the phase function of a wide range of media, in rendering the image, it is necessary nevertheless to measure other scattering characteristics such as the coefficients of extinction and scattering. Our scattering model presumes these coefficients are known. The objective, which remains as an open problem, is to measure the scattering properties of a wide range of media and to summarize them all in a single data set.

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REFERENCES

Ioannis Gkioulekas, Bei Xiao, Shuang Zhao, Edward H. Adelson, Todd Zickler, and Kavita Bala. 2013a. Understanding the Role of Phase Function in Translucent Appearance. *ACM Trans. Graph.* 32, 5, Article 147 (Oct. 2013), 19 pages. https://doi.org/10.1145/2516971.2516972

Ioannis Gkioulekas, Shuang Zhao, Kavita Bala, Todd Zickler, and Anat Levin. 2013b. Inverse Volume Rendering with Material Dictionaries. *ACM Trans. Graph.* 32, 6, Article 162 (Nov. 2013), 13 pages. https://doi.org/10.1145/2508363.2508377

Tim Hawkins, Per Einarsson, and Paul Debevec. 2005. Acquisition of Time-varying Participating Media. *ACM Trans. Graph.* 24, 3 (July 2005), 812–815. https://doi.org/10.1145/1073204.1073266

L.G. Henyey and J.L. Greenstein. 1941. Diffuse radiation in the Galaxy. *Astrophysical Journal* 93 (Jan. 1941), 70–83. https://doi.org/10.1086/144246

Henrik Wann Jensen, Stephen R. Marschner, Marc Levoy, and Pat Hanrahan. 2001. A Practical Model for Subsurface Light Transport. (2001), 511–518. https://doi.org/10.1145/383259.383319

Srinivasa G. Narasimhan, Mohit Gupta, Craig Donner, Ravi Ramamoorthi, Shree K. Nayar, and Henrik Wann Jensen. 2006. Acquiring Scattering Properties of Participating Media by Dilution. *ACM Trans. Graph.* 25, 3 (July 2006), 1003–1012. https://doi.org/10.1145/1141911.1141986