

# Layered optical tomography of multiple scattering media with combined constraint optimization

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## 1. Introduction

In this paper, we describe a method to solve the optical tomography by using an approximation to the path integral to model the light in optical tomography. Since X-ray CT relies on radioactive rays, optical tomography uses visible or infrared ray has been developed over the last decades[1, 2] for its safety and harmless. Our research aims to develop an optical tomography method that use infrared or near infrared ray input and observed outgoing light at the opposite side of the body, shown as in Figure 1(a), like as a source-detector configuration that X-ray CT uses.

In our previous work[3], we use the concepts of path integral and light transport to model the light in optical tomography and solved the tomography problem.

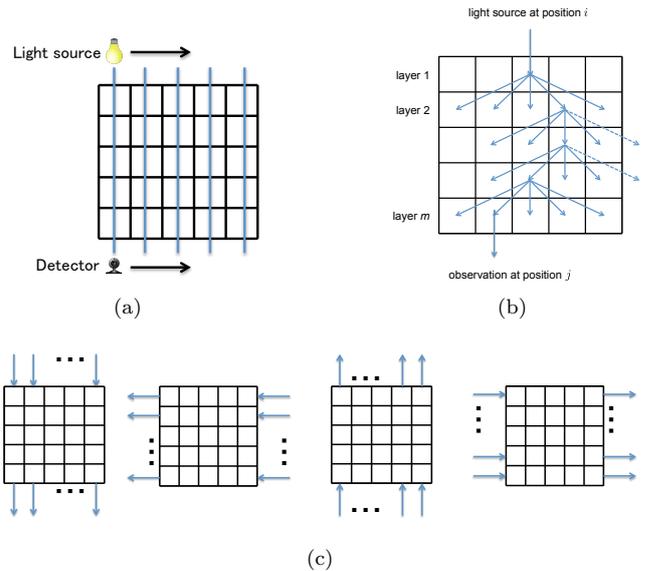
In this paper, we extended our previous work[3]. By using multiple configuration, as shown in Figure 1(c), we increase the accuracy and by changing the solver we reduce the computation cost successfully.

In our previous work[3], we took the following assumptions (see Fig. 1(b)) : (1) multiple scattering is dominant, (2) forward scattering is also dominant relative to backward scattering, (3) a material consists of many parallel layers made of voxels, and (4) light is scattered from one layer to another because forward scattering is assumed be dominant. The material follow the assumptions above is the layered material. With the layered material, we developed a constraint optimization problem to solve the optical tomography.

The structure of rest part of the paper is listed as following. In section 2, we review the method in our previous work and show the changes. In section 3, we show the simulation results and compare them with our previous work. Section 4 gives our conclusion.

## 2. Method

In our previous work[3], we proposed a model to describe the transportation of light penetrate through the material which follows the assumptions in last section. We'll briefly review it in next paragraph.



**Fig. 1** Configurations of light sources and observations. (a) Source-detector configuration of CT. (b) A single configuration with the layers model. A light source at position  $i$  emits light to the first layer, then the light is scattered to the next layer. At the last layer, output is observed at each position  $j$ . (c) Four configurations. The object is fixed while the light source and detector are rotated by 90 degrees.

In our previous work[3], in a  $M$  by  $N$  layered material, given an incident point  $i$  and the camera point  $j$ , the observed light intensity  $I_{ij}$  obtained by model in [3] is shown as following

$$I_{ij} = \sum_{k=1}^{M^{N-2}} I_{ijk} = \sum_{k=1}^{M^{N-2}} I_0 c_{ijk} a_{ijk}, \quad (1)$$

Since the observed intensity is affected by all possible light path that come in point  $i$  and observed in point  $j$ , the number of all possible light path is  $M^{N-2}$  and intensity for every light path is  $I_{ijk}$ . For a single light path  $ijk$ , its intensity is affected by 3 parts, intensity of incident light which is  $I_0$ , intensity lost due to the scattering which is  $c_{ijk}$  and intensity lost due to the attenuation which is  $a_{ijk}$ . In the layered material,  $c_{ijk}$  is obtained by Gaussian model[4] and its magnitude is control by  $\sigma^2$ .  $\sigma^2$  describe how broad the light is scattered.  $a_{ijk}$  describe the exponential attenuation when light path  $ijk$  penetrate through the layered material and it has the following formation

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$$a_{ijk} = e^{-\mathbf{d}_{ijk}^T \boldsymbol{\sigma}_t}. \quad (2)$$

Here  $\mathbf{d}_{ijk}$  describe the light path  $ijk$  and  $\boldsymbol{\sigma}_t$  present the extinction coefficient of the voxels in the layered material.

By keeping light source at the top of material, camera at the bottom of material and then changing a pair of  $(i, j)$ , the positions of incident and outgoing points of light, we have  $M^2$  observations  $I_{ij}$  and equations to solve the following least squares problem:

$$\min_{\boldsymbol{\sigma}_t} f_{tp}, \quad f_{tp} = \sum_{i=1}^m \sum_{j=1}^m |I_{ij} - \sum_{k=1}^{m-2} I_0 c_{ijk} e^{-\mathbf{d}_{ijk}^T \boldsymbol{\sigma}_t}|^2, \quad (3)$$

Once we have  $f_{tp}$ , we can rotate the material 90 degrees for 3 times to obtain  $f_{lr}, f_{pt}, f_{rl}$ . Then the final object function  $f_0$  is

$$\min_{\boldsymbol{\sigma}_t} f_0, \quad f_0 = f_{tp} + f_{lr} + f_{pt} + f_{rl} \quad (4)$$

Due to the fact that the extinction coefficient must be positive and the numerical stability, we constrain Equ.4 with  $0 \preceq \boldsymbol{\sigma}_t \preceq u$ .  $\preceq$  denotes generalized inequality that every elements in a vector must satisfy the inequality

In our previous work[3], we use interior point method cooperated with Newton's method to solve the optimization problem obtained from the optical tomography. Since Newton's method employ second derivative of the cost function, it is not the best choice for large scale material. Therefore, in this paper, we replace the Newton's method with Quasi-Newton's method and keep parameters same with [3]. Quasi-Newton's method uses first derivative of the cost function to approximate to second derivative which can greatly reduce the computation time when dealing with large scale material.

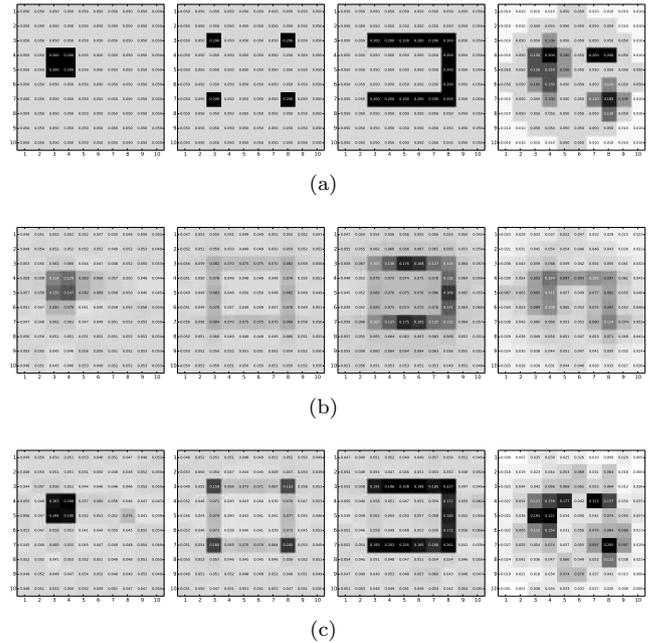
### 3. Experiment

We evaluate the proposed method by numerical simulation. Four kinds of materials of the size  $10 \times 10$  shown in Figure 2(a) are used. Each material has almost homogeneous extinction coefficients (in light gray) except few voxels with much higher coefficients (in darker gray), which means those voxels absorb light much more than others. Parameters are set as follows:  $\sigma^2 = 1.0$  for scattering;  $u = 1.0$  for the upper bound.

Estimated results are shown in Figure 2(b) and (c): results in the second row (b) are obtained by previous work [3], while results in the third row (c) are by our proposed method. Our results (c) are much closer to the ground truth (a) and better than the previous work (b). This is also confirmed by qualitative results in terms of root mean squares error (RMSE) shown in Table 1. Table 2 shows computation time spent by the previous work and proposed method; it is roughly reduced to 30% (by our unoptimized code in MATLAB on a PC with Intel Xeon E5 2GHz).

### 4. Conclusion

In this paper, we have proposed an improved scattering to-



**Fig. 2** Simulation results for  $\sigma^2 = 1.0$ . (a) Ground truth of four materials 1, 2, 3, and 4. (b) Results of [3]. (c) Results of our method. Values in each voxel are estimated value of  $\boldsymbol{\sigma}_t$ , and darker gray represents larger value.

**Table 1** RMSEs of results for four materials. The order is the same with Figure 2(b) and (c).

method	1	2	3	4
Fig. 2(b) [3]	0.016978	0.02731	0.043248	0.030447
Fig. 2(c) ours	0.004987	0.01213	0.010154	0.022141

**Table 2** Computation time (in seconds) of results for four materials in Figure 2(b) and (c).

method	1	2	3	4
Fig. 2(b) [3]	191.334	162.035	197.519	195.628
Fig. 2(c) ours	55.602	55.602	70.531	76.773

mography with a layered model. We modified the optimization problem we proposed in our previous work[3]. Then we solved it by interior point method with a Quasi-Newton method. In the experiments, results were much more improved than the previous work as well as computational cost is decreased.

### References

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