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Mirror Localization for a Catadioptric Imaging System by Projecting Parallel Lights

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***Abstract*— This paper describes a method of mirror localization to calibrate a catadioptric imaging system. Even though the calibration of a catadioptric system includes the estimation of various parameters, in this paper we focus on the localization of the mirror. Since some previously proposed methods assume a single view point system, they have strong restrictions on the position and shape of the mirror. We propose a method that uses parallel lights to simplify the geometry of projection for estimating the position of the mirror, thereby not restricting the position or shape of the mirror. Further, we omit the translation process between the camera and calibration objects from the parameters to be estimated by observing some parallel lights from a different direction. We obtain the constraints on the projection and compute the error between the model of the mirror and the measurements. The position of the mirror is estimated by minimizing the error. We also test our method by simulation and real experiments, and finally we evaluate the accuracy of our method.**

I. Introduction

Catadioptric imaging systems are often used to obtain various fields of view by observing rays reflected from mirrors. In particular, catadioptric omnidirectional imaging systems [1], [2], [3] are widely used in various applications, such as robot navigation, surveillance, and virtual reality.

There are two types of catadioptric imaging systems: central and noncentral. The former has a single effective viewpoint, and the latter has multiple viewpoints. Although central catadioptric systems have the advantageous feature that the image can be transformed to a perspective projection image, they have strong restrictions on the shape and position of the mirror. For example, it is necessary to use a telecentric camera and a parabolic mirror whose axis is aligned to the axis of the camera. Thus, a catadioptric system may not be a central due to possible misconfiguration. Several noncentral systems [4], [5], [6], [7] have been proposed for various purposes.

For geometric analysis with catadioptric systems, it is necessary to calibrate both the camera and mirror parameters. Several methods of calibration have been proposed for central catadioptric systems. Geyer and Daniilidis [8] used three lines to estimate the focal length, mirror center, etc. Ying and Hu [9] used lines and spheres to calibrate the parameters. However, since these methods assume that the system has a single viewpoint, they cannot be applied to noncentral systems. On the other hand, several methods have also been proposed to calibrate noncentral imaging systems. Aliaga [10] estimated the parameters of a catadioptric system with a perspective camera and a parabolic mirror using known 3D points. Strelow et al.[11] estimated the position of a misaligned mirror using known 3D points. Micusík and Pajdla [12] fitted an ellipse to the contour of the mirror and calibrated a noncentral camera by approximating it to a central camera. Mashita et al. [13] used the boundary of a hyperboloidal mirror to estimate the position of a misaligned mirror. However, these methods are restricted to omnidirectional catadioptric systems.

There are also some approaches for calibrating more general imaging systems. Swaminathan et al. [14] computed the parameters of noncentral catadioptric systems by estimating a caustic surface from known camera motion and the point correspondences of unknown scene points. Grossberg and Nayar [15] proposed a general imaging model and computed the ray direction for each pixel using two planes. Sturm and Ramalingam [16] calibrated the camera of a general imaging model by using unknown camera motion and a known object. Pless [17] estimated the position of a multi-camera system based on structure from motion. Since these methods estimate both the internal and external parameters of the system, the error of measurement affects the estimated result of all of the parameters.

In this paper, we focus on the localization of the mirror in the calibration of catadioptric systems. The other parameters are assumed to be as follows:

- The internal parameters, such as the focal length and principal point of a camera, are known.
- The shape of the mirror is known.
- Translation to the world coordinate system is omitted from the calibration.

When we create a mirror to use in a catadioptric system, we know the shape of the mirror with the accuracy of its cutting or molding machine. Therefore the knowledge of the mirror shape is not a strong assumption. By omitting translation from the calibration, we reduce the number of parameters that are affected by the measurement error in the calibration.

To omit translation from the calibration, our method uses parallel lights as calibration patterns. Since a parallel light is regarded as a ray projected from an object at infinite distance, such as the sun, we do not have to consider the translation from a calibration pattern to the camera. In Section II, we describe the difference of the geometry of projection of

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a point light source and a parallel light. Next, in Section III, we propose an algorithm for mirror localization using parallel lights. We test our method in Section IV and finally summarize this paper in Section V.

II. Geometry of Projection

In this section, we explain the difference between the projection of a point light source and that of a parallel light.

A. Projecting A Point Light Source

If an object or feature point is projected to an imaging system, it is regarded as the projection of a point light source at a finite distance. Thus, the calibration object is usually considered to be a point light source. The 3D position of the point light source is $\boldsymbol{p}$. The ray from the point light source is reflected by the mirror at $\boldsymbol{x}$. The reflected ray arrives at the viewpoint of camera $\boldsymbol{O}$ through image point $\boldsymbol{m}$. Since the reflection angle at a mirror is equal to the incident angle,

$$\frac{\boldsymbol{p}-\boldsymbol{x}}{\parallel \boldsymbol{p}-\boldsymbol{x} \parallel} - \frac{\boldsymbol{m}}{\parallel \boldsymbol{m} \parallel} = aN_{R,\boldsymbol{t}}(\boldsymbol{x}), \tag{1}$$

where $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ is the normal vector of the mirror surface at point $\boldsymbol{x}$. R and $\boldsymbol{t}$ are the rotation and translation of the mirror, respectively, and a is a scale factor. By removing scale factor a from (1), we obtain two equations.

We consider the following three cases of unknown parameters:

1) The 3D positions of points are completely unknown.
2) The relative positions between points are known, while their relative positions to the camera are unknown.
3) Their relative positions to the camera are known.

The first case corresponds to structure from motion that simultaneously estimates the position of the camera and the position of 3D points. If position $\boldsymbol{p}$ is unknown, then the parameters to be estimated are R, $\boldsymbol{t}$, R_C, $\boldsymbol{t}_C$ and $\boldsymbol{p}$ of each point, where R_C and $\boldsymbol{t}_C$ are the rotation and translation parameters, respectively, of the camera for each image. When n points are observed by k images by changing viewpoints, the number of parameters is $6+6k+3n$, and the number of equations is $2kn$. Thus, k and n should satisfy $6+6k+3n \leq 2kn$. The minimum number of parameters is 48 if $k=3, n=8$ or $k=4, andn=6$. In this case, therefore, even though the parameters that we really want to obtain are R and $\boldsymbol{t}$ only, the number of parameters to be estimated is too great.

The second case corresponds to calibration using a structured calibration object, such as a checker board, lines, and circles. If the relative positions of the 3D points are known, then the parameters to be estimated are R, $\boldsymbol{t}$, R_C, and $\boldsymbol{t}_C$. Though the number of parameters is reduced to $6+6k$, where k is the number of images, the number of constraints varies according to both the calibration object and the projection model. Further, though the minimum number of parameters is 12, since we do not assume single viewpoint projection models, the actual number becomes larger than 12 in order to obtain constraints using a calibration object for general catadioptric imaging systems.

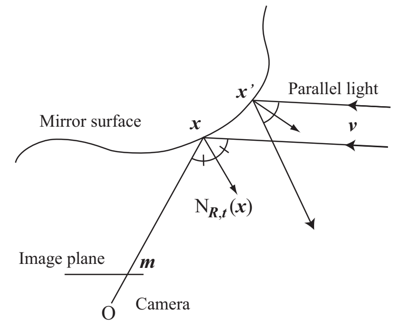


Fig. 1. Projecting a parallel light to a catadioptric imaging system.

In the third case, the positions of the 3D points relative to the camera origin are measured a priori. If the position of point light source $\boldsymbol{p}$ is known, that is, if the position of the calibration object is known in the camera coordinate system, then the parameters to be estimated are only R and $\boldsymbol{t}$. The number of parameters is therefore reduced to 6. However, position $\boldsymbol{p}$ is difficult to measure in an actual case since it is in a position relative to camera origin $\boldsymbol{O}$, which is unknown in the world coordinate system. Hence, the accuracy of this approach is poor.

B. Projecting a Parallel Light

In this section, we explain the geometry of projecting a parallel light and the reduction of parameters to be estimated. Figure 1 shows the projection of a parallel light. The difference here compared to projecting a point light source is that the light source becomes a parallel light whose direction is $\boldsymbol{v}$. Since a parallel light is equal to a light from a distant point light source, $\boldsymbol{v}$ is regarded as

$$\boldsymbol{v} = -\lim_{s\to\infty} \frac{s\boldsymbol{p}-\boldsymbol{x}}{\parallel s\boldsymbol{p}-\boldsymbol{x} \parallel}. \tag{2}$$

Thus, the equation of projection becomes

$$-\boldsymbol{v} - \frac{\boldsymbol{m}}{\parallel \boldsymbol{m} \parallel} = aN_{R,\boldsymbol{t}}(\boldsymbol{x}), \tag{3}$$

where $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ is the normal vector of the mirror surface at point $\boldsymbol{x}$. R and $\boldsymbol{t}$ are the rotation and translation of the mirror, and a is a scale factor. Similar to (1), we obtain two equations by removing scale factor a from (3).

If we use a parallel light, the translation from the light source to the camera can be removed from the parameters. Since vector $\boldsymbol{v}$ is a relative direction in the camera coordinate system, $\boldsymbol{v}$ actually consists of two parameters. Therefore, when n different parallel lights are observed by k images by rotating the camera, the number of parameters is $6+3k+2n$.

A method we propose in Section III, which estimates the position of the mirror, uses turntables to rotate the camera. While the translation is difficult to measure, the rotation parameter of a turntable is accurately measured by using an encoder. Because the relative directions between parallel lights of different rotations are known, the other unknown parameters are the rotation of the camera and the direction

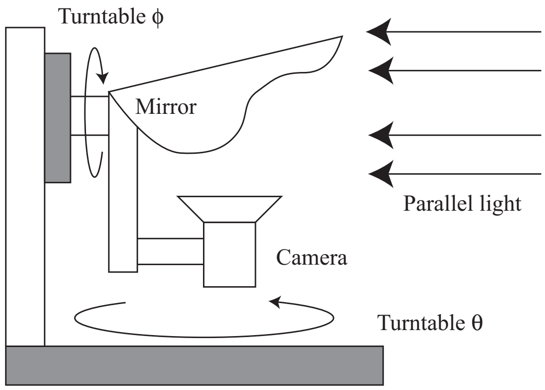


Fig. 2. Calibration system using parallel lights: The camera and mirror is rotated by two turntables.

of a parallel light. Therefore, the number of parameters is reduced to $6 + 3 + 2$.

Moreover, our method independently computes the direction of the parallel light from an estimation of the mirror position. Therefore, we separate the problem into two steps. The first estimates the direction of the parallel light and the second estimates the mirror position. The numbers of parameters for the two steps are 2 and 9. This is much smaller than that in the case of point light sources.

III. Mirror Localization using Parallel Lights

This section describes an algorithm to estimate mirror position. Our method consists of two steps:

1) Estimating the direction of a parallel light
2) Estimating the mirror position

We estimate the direction of a parallel light and the mirror position separately.

Figure 2 shows our calibration system. Since the camera and mirror are mounted on two turntables and rotated by them, they rotate around two axes that are perpendicular to each other. Thus, the relationship between the camera and the mirror does not change with the rotation. To project a parallel light, we use a distant point object or a concave parabolic mirror to generate a parallel light. If we use a parabolic mirror, a point light source is set at the focal point of the parabolic mirror, and the reflected light becomes a parallel light. The catadioptric imaging system obtains parallel lights from various directions while rotating.

A. Estimating Parallel Light Direction

When a rotating camera acquires images of the parallel light, some projected points are in the same position even if the rotation parameters are different. Namely, if two images are obtained with the rotation parameters of turntables R_{T1} and R_{T2} ($R_{T1} \neq R_{T2}$), then the projected points $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ may become $\boldsymbol{m}_1 = \boldsymbol{m}_2$. This is because the rotation of the camera in this system has ambiguity concerning the rotation around the ray vector. In an actual experiment, we find such a pair, $T1$ and $T2$, by rotating the turntables.

If $\boldsymbol{m}_1 = \boldsymbol{m}_2$, then the ray vector in the camera coordinate system $\boldsymbol{v}$ is the same for each rotation. Thus,

$$R_C R_{T1} \boldsymbol{v}_0 = R_C R_{T2} \boldsymbol{v}_0, \qquad (4)$$

where the ray vector in the world coordinate system is $\boldsymbol{v}_0$ and R_C is the rotation of the camera relative to the turntables. Since a parallel light is projected, the translation from the center point of rotation to the camera origin is omitted from the equation. By modifying (4) as

$$(R_{T1} - R_{T2}) \boldsymbol{v}_0 = 0, \qquad (5)$$

$\boldsymbol{v}_0$ is computed as the eigenvector of $M^T M$ associated with the smallest eigenvalue, where $M = R_{T1} - R_{T2}$. Though $\boldsymbol{v}_0$ can be computed by a pair of R_{T1} and R_{T2}, if we obtain some pairs of projected points, we can create M by stacking all rows of $R_{T1} - R_{T2}$ and solve $M\boldsymbol{v}_0 = 0$ similarly.

Our method observes the projected point of a parallel light by rotating the camera, and it finds the projected point where the parallel light is projected with different rotation parameters. If the rotation matrices of two turntables are R_θ and R_ϕ, respectively, then a rotation parameter is computed as

$$R_T = R_\theta R_\phi. \qquad (6)$$

The simplest way to compute the light direction is to set the incoming light parallel to the axis of turntable θ or ϕ. In this case, $\boldsymbol{v}_0$ is easily obtained as the axis vector. If we use a distant point object as a parallel light source, we can find a point that does not move in the image while rotating by turntable θ or ϕ, and then use that point as the projection of a parallel light.

B. Estimating Mirror Position

We estimate the mirror position by minimizing the following functional:

$$\sum_{\boldsymbol{m}} \parallel N_{R,\boldsymbol{t}}(\boldsymbol{x}) - \boldsymbol{n} \parallel^2, \qquad (7)$$

where $\boldsymbol{n}$ is a normalized vector of $-\boldsymbol{v} - \boldsymbol{m}/||\boldsymbol{m}||$, and $||N_{R,\boldsymbol{t}}(\boldsymbol{x})|| = 1$. Since minimizing (7) is a nonlinear minimization problem, we estimate R, $\boldsymbol{t}$, and R_C by the Levenberg-Marquardt algorithm. Our algorithm becomes :

1) Set the initial parameters of R, $\boldsymbol{t}$, and R_C.
2) Compute intersecting point $\boldsymbol{x}$ for each image point $\boldsymbol{m}$.
3) Compute the error function (7).
4) Update R, $\boldsymbol{t}$, and R_C by the Levenberg-Marquardt algorithm.
5) Repeat steps 2-4 until convergence.

We explain computing $\boldsymbol{n}$ and $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ in the rest of this section.

Once the direction of parallel light $\boldsymbol{v}_0$ is computed, the ray vector $\boldsymbol{v}$ for each rotation parameter R_T in the camera coordinate system is computed as

$$\boldsymbol{v} = R_C R_T \boldsymbol{v}_0. \qquad (8)$$

Since we assume that the internal parameters of the camera are known, $\boldsymbol{m}/||\boldsymbol{m}||$ is computed from the projected pixel of

TABLE I
ERROR OF ESTIMATED PARALLEL LIGHT DIRECTION: THE MEAN ANGULAR ERROR AND THE STANDARD DEVIATION (STD.) OF THE ERROR (DEGREE).

σ	1 pair		6 pairs	
(pixels)	Mean	Std.	Mean	Std.
0.0	0.008	0.004	0.005	0.001
0.1	0.053	0.030	0.024	0.014
0.5	0.205	0.104	0.078	0.038

the parallel light. Thus, $\boldsymbol{n}$ is computed from the observation of the parallel light and turntables.

Since $\boldsymbol{x}$ is the intersecting point of viewing vector $\boldsymbol{m}$ and the mirror surface, normal vector $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ is represented as a function of R, $\boldsymbol{t}$, and $\boldsymbol{m}$. Because we do not assume the shape of the mirror, it is generally difficult to compute $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ by algebraic manipulation. Therefore, we compute $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ numerically in this paper. To accommodate any mirror shape, we approximate the mirror shape by a mesh model. Intersecting point $\boldsymbol{x}$ is computed by projecting the mesh model onto the image plane of the camera with R, $\boldsymbol{t}$, and the internal parameters of the camera. $N_{R,\boldsymbol{t}}(\boldsymbol{x})$ is computed as a normal vector of the mesh model at $\boldsymbol{x}$.

IV. EXPERIMENTS

A. Estimating Accuracy by Simulation

We first evaluate the accuracy of our method by simulation. In this simulation, we estimate the position of a parabolic mirror relative to a camera. Since our method consists of two steps, we evaluate each step separately.

For the first step, we measure the accuracy of the estimated parallel light direction. In this experiment, the camera is a perspective camera that has a 40° field of view. The size of the image is 640×480 pixels. Radius h of the parabolic mirror is 2.5, where the shape is defined as $z = \frac{1}{2h}r^2, (r^2 = x^2 + y^2)$. The diameter and height of the mirror are 10 and 5, respectively. A mesh model of the mirror is created by approximating a paraboloid with triangles of length 0.05 on a side. The mirror is shifted by $(0.3, -0.6, 18.0)$ along each axis and rotated by $(1.2, -0.8, 0.0)$ (degrees) around each axis. We measured the error of the estimated parallel light direction by adding a noise on the projected position of the parallel light in an image. Table I shows the angular error of the estimated parallel light direction. We added a Gaussian noise of $N(0, \sigma^2)$ to the projected position. The angular error is the angle between the vector of ground truth and the estimated result. We compared the results computed using 1 pair and 6 pairs of projected points and found that the error can be reduced if we use 6 pairs.

For the second step, we estimate the accuracy of the estimated mirror position. In this experiment, the camera is a perspective camera with an image of 512×512 pixels and focal length of 900 pixels. Radius h of the parabolic mirror is 9.0. The diameter and height of the mirror are 25.76 and 9.0, respectively. The mirror is shifted by $(0.0, 0.0, 50.0)$ along each axis. Thus, this catadioptric imaging system is not a single viewpoint system. To validate the accuracy of our method, we compared the three methods by changing the number of parameters, as shown in Table II. Since Method 1 and 2 use point light sources as feature points, they must estimate the camera positions together with the mirror position. Method 2 also estimates the 3D positions of the feature points, while Method 1 assumes that the relative positions of the feature points are known. Therefore, Method 1 corresponds to calibration using a structured calibration object, and Method 2 corresponds to structure from motion. In Methods 1 and 2, we minimized (7) by changing $\boldsymbol{n}$ to

$$\frac{\frac{\boldsymbol{p}'-\boldsymbol{x}}{\|\boldsymbol{p}'-\boldsymbol{x}\|} - \frac{\boldsymbol{m}}{\|\boldsymbol{m}\|}}{\| \frac{\boldsymbol{p}'-\boldsymbol{x}}{\|\boldsymbol{p}'-\boldsymbol{x}\|} - \frac{\boldsymbol{m}}{\|\boldsymbol{m}\|} \|}, \tag{9}$$

where $\boldsymbol{p}' = R_C\boldsymbol{p} + \boldsymbol{t}_C$, and R_C and $\boldsymbol{t}_C$ are the translation and rotation parameters, respectively, of the camera for each image. In Method 2, the positions of 3D feature points $\boldsymbol{p}$ are also parameters.

We measured the error of the estimated mirror position by adding Gaussian noises on the projected positions of the features, which were $\sigma = 0.0, 0.1, 0.5, 1.0$ (pixels). For our method, we also added noises on the parallel light direction to compute the mirror position. We use the mean error shown in Table I as the magnitude of noise when the light direction is estimated by 6 pairs of projected points. We assume that an initial value is known. Figure 3 shows the results of the error of the estimated mirror position. We tested our method with 6 and 24 feature points to change the number of the constraints. We compute the RMS error of the translation between the ground truth and the estimated position in Figure 3(a). In Figure 3(b), we estimate the reprojected points of the incoming lights using the estimated position of the mirror. Since the error of the proposed method is one-half and one-quarter of that of Method 1 and Method 2, respectively, the result of our method is better than that of other methods.

B. Localizing Mirrors from Real Images

To localize a mirror from real images, we created an experimental calibration system with two turntables, as shown in Figure 4. A camera and multiple mirrors are mounted on the system and rotated by the turntables.

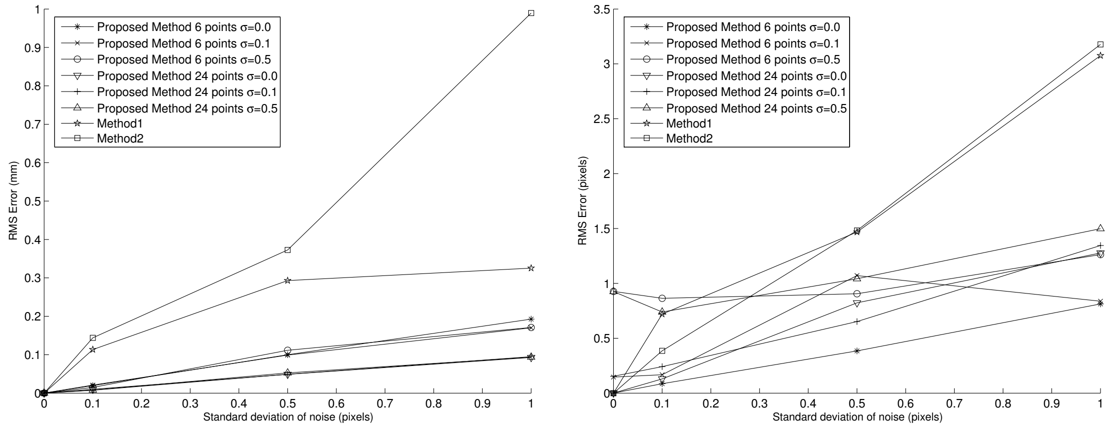
In this experiment, we computed the mirror positions of a catadioptric system with compound parabolic mirrors, which has been proposed in [18], [19], [20]. Figure 5 shows an example of the image. The system has 7 parabolic mirrors, and the camera is a perspective camera. The camera is a PointGrey Scorpion, which has 1600×1200 pixels and about a 22.6° field of view. The distortion of the lens is calibrated by the method [21], and the internal parameters of the camera are computed using OpenCV [22] as a preprocessing of calibrating the mirrors. In this setup, the catadioptric system is not a single viewpoint system.

The radii h of the center mirror and side mirrors are 9.0mm and 4.5mm, respectively. The diameter and height of the center mirror are 25.76mm and 9.0mm, respectively, and the diameter and height of the side mirrors are 13.0mm and

TABLE II

COMPARISON OF THREE METHODS FOR ESTIMATING THE MIRROR POSITION. METHODS 1 AND 2 USE POINT LIGHT SOURCES AS FEATURE POINTS. IN METHOD 1 THE RELATIVE POSITIONS BETWEEN POINTS ARE KNOWN. IN METHOD 2 THE POSITIONS OF POINTS ARE COMPLETELY UNKNOWN.

Method	# of mirror parameters	# of camera positions	# of features	# of external parameters	# of param. of features	total # of parameters	total # of constraints
Our method	6	1	6	3	0	9	12
	6	1	24	3	0	9	48
Method 1	6	4	6	24	0	30	48
Method 2	6	4	6	24	18	48	48



(a) (b)

Fig. 3. Error in estimating the mirror position: (a) RMS error of the translation. (b) RMS error of the reprojected points.

4.5mm, respectively. The diameters of the center and side mirrors projected onto the image are 840 and 450 pixels, respectively.

We used a distant point as a parallel light source. We first chose a point in the image that does not move while rotating Turntable 2. Figure 6 shows the chosen point, which is a point of a building that is about 260 meters away from the camera. Then, we rotated the two turntables and found the point in the images manually. We obtained 13 lights from various directions.

We estimated the positions of the center and four side mirrors independently. Table III shows the estimated results. Since some of lights are occluded by the other mirrors, the number of lights used for calibration varies depending on the mirror positions. The side mirrors are designed to be put at the Designed position relative the center mirror. The Estimated positions are the positions relative to the camera computed by the proposed method. The Relative positions are the estimated positions relative to the center mirror. We compared the Designed and Relative positions. The errors along the x- and y-axes, which are parallel to the image plane, were less than 1mm. Though the errors along the z-axis are about 1mm, we believe that this is because the field of view of the camera is narrow. To evaluate the accuracy in the image space, we reprojected the lights to the image using the estimated mirror positions. The Reprojection errors are the distances of the reprojected points from the input points. Since we chose the input points manually, the error of an input point is about 1 pixel. Therefore, it is reasonable to conclude that the reprojection error is about 1 pixel, and therefore that the proposed method works well.

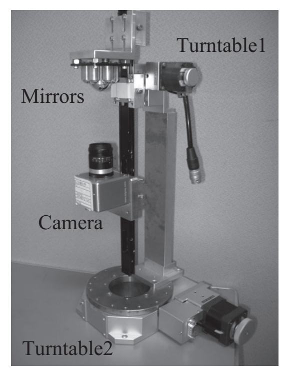


Fig. 4. Experimental calibration system with two turntables.

V. CONCLUSION

This paper described a method of mirror localization to calibrate a catadioptric imaging system. In particular, we focused on the localization of the mirror. Our method used parallel lights to simplify the geometry of the projection. Since translation between the camera and the calibration objects is omitted from the parameters, the number of parameters to be estimated is reduced. By observing some parallel lights from different directions, we could obtain the constraints on projection and compute the error between the model of the mirror and the measurements. Our method separately estimates the parallel light direction and the mirror

TABLE III

THE ESTIMATED MIRROR RESULTS: THE SIDE MIRRORS ARE DESIGNED TO BE PUT AT THE DESIGNED POSITION RELATIVE THE CENTER MIRROR. THE ESTIMATED POSITIONS ARE THE POSITIONS RELATIVE TO THE CAMERA COMPUTED BY THE PROPOSED METHOD. THE RELATIVE POSITIONS ARE THE ESTIMATED POSITIONS RELATIVE TO THE CENTER MIRROR. WE COMPARED THE DESIGNED AND RELATIVE POSITIONS TO COMPUTE THE POSITION ERRORS. THE REPROJECTION ERRORS ARE THE DISTANCES OF THE REPROJECTED POINTS FROM THE INPUT POINTS USING THE ESTIMATED MIRROR POSITIONS.

Mirror	Number of Lights	Position (mm)			Error	
		Designed	Estimated	Relative	Position (mm)	Reprojection (pixels)
Center	13	(0, 0, 0)	(-0.07, 1.41, 75.40)	(0, 0, 0)	N.A	2.24
Side1	7	(-15.00, 0, 9.22)	(-15.46, 2.02, 85.99)	(-15.39, 0.61, 10.59)	(-0.39, 0.61, 1.37)	0.92
Side2	10	(-7.50, -12.99, 9.22)	(-7.79, -11.21, 85.10)	(-7.72, -12.62, 9.70)	(-0.22, 0.37, 0.48)	1.67
Side3	10	(7.50, -12.99, 9.22)	(7.50, -11.29, 85.52)	(7.57, -13.33, 10.12)	(0.07, -0.23, 0.90)	1.38
Side4	7	(15.00, 0, 9.22)	(15.21, 1.56, 86.02)	(15.28, 0.15, 10.62)	(0.28, 0.15, 1.40)	0.76

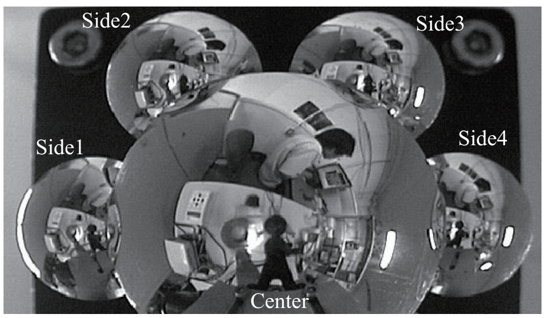


Fig. 5. Example of the image of compound parabolic mirrors.

Fig. 6. A distant point is used as a parallel light source.

position. Finally, to validate the accuracy of our method, we tested our method in both a simulation and in real experiments. In future studies, we will work on automatically finding the parallel lights with the aim to improve the accuracy of the mirror localization.

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