Estimating Parameters of Subsurface Scattering using Directional Dipole Model

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***Abstract*—Acquisition of parameters for the Bidirectional Scattering Surface Reflectance Distribution Function (BSSRDF) have significant meanings in the study of Computer Graphics and Vision research field. In this paper, we present an inverse rendering approach combined with a newly developed BSSRDF model (directional dipole model) for parameter estimation of spherical surface. To validate our algorithm, we estimate parameter from spheres with a wide range of radius on simulated and real environment, respectively. According to the observation from both simulated and real experiments, we find that surface curvature significantly affect the estimation result.**

***Keywords*-subsurface scattering, inverse rendering, sphere, computer graphics**

I. INTRODUCTION

To synthesize realistic image of translucent material, lots of Bidirectional Scattering Surface Reflectance Distribution Function (BSSRDF) [1], [2], [3] have been proposed to simulate subsurface scattering. Knowing the parameters for these BSSRDF model, is not only necessary to render realistic translucent materials, but also have significant meaning in studies of computer vision (CV) like shape acquisition[4].

Parameter acquisition approaches for the BxDF (e.g., BSSRDF) family have appeared in as early as 1999. To estimate parameters of Bidirectional Reflectance Distribution Function (BRDF), inverse global illumination method[5] was proposed in 1999. Inspired by their work, lots of works to estimate parameters for BSSRDF [1], [6], [2], [7], [4] have been done over the past decades. Whereas, due to the complicated way in which translucent material interact with light, most previous work based on the condition of planar, or globally planar surface and parameter estimation from non-planar surface remains a considerable challenge. As one step towards the solution of inverse rendering problem of arbitrary non-planar surface, our work focus on estimating scattering parameters from spherical surface. Motivated by theory presented in [8], which shows that the effect of subsurface scattering is strongly correlated to radius of the sphere, we assume that the radius of sphere also significantly affect the acquisition of parameters from sphere. In this paper, while we do not explicitly give a parameter estimation approach considering the radius of sphere, we make some progress in this direction by finding a suitable analytical BSSRDF capable of estimation from spherical surface and verifying the correlation between sphere curvature and estimation result.

We conclude our work provide a method to estimate BSSRDF parameter from spherical surface and is a base for further estimation of object with more complicated shape.

II. RELATED WORK

A. Analytical BSSRDF

To simulate subsurface scattering, conventional path tracing algorithm traces very explicitly the transport path of photon inside the material and therefore suffers from the high computational cost. For more practical rendering of translucent material, approximation models for subsurface scattering are developed to simplify the computation. Early in [1], Jensen et al. introduced a dipole approximation based on diffusion theory. It achieved a plausible trade-off between correctness and efficiency. Though limited to assumption of semi-infinite planar surface and homogeneous material, dipole is still one of the most widely employed BSSRDF to synthesize translucent material. For the application in more complicated geometries, BSSRDFs are derived from the dipole model, such as multipole[9], [10] for multilayered slib and quadpole[11] for right-angle corner, respectively. In [3], Frisvad et al. introduced a new promotion of the dipole family. On the base of dipole, the dipole point source was replaced with a set of directional dipole source to meet the boundary condition of diffusion equation. This new configuration allows their model to formulate the single scattering component into their model. Experiments showed directional dipole model can have a closer prediction to results computed using unbiased path tracing, with relatively better efficiency. All these previous work about modeling subsurface scattering offer us a foundation for the analysis of the subsurface scattering.

B. Parameter estimation using inverse rendering

While forming image from a specified scene is called a rendering procession, techniques used to acquire scene information from image are known as inverse rendering. Such kind of techniques for parameter estimation can be traced back to work of Yu in 1999[5]. In this work, they first derived a rendering equation for the scene and recovered the surface reflectance and albedo by minimizing the error between observation and simulation. Despite the limitation of opaque material, the basic idea of their work is seen in later studies. Along with the introduction of dipole model, Jensen et al. also acquired parameters for their model by illuminating an optical thick translucent planar surface with collimated beam and searching parameters best explain observation in [1]. In [4], Dong et al. estimated scattering parameters and surface normal map for globally planar surface simultaneously by solving a non-liner optimization problem iteratively. To evaluate their estimation of parameters, approach similar with Jensen et al.[1] was applied as baseline measurement in their work. Donner et al. specialized the multipole model to better express human skin in [2]. Similarly, they setup a special environment and derived an image formation model for their scene. For the high freedom degree of their model, they chose to solve the optimization problem by utilizing photographs observed under different wavelength. One common ground of these work is that they focus on only a single face of the target sample.

Base on the single scattering approximation, Narasimhan et al. acquired scattering parameters of low concentration participating liquid media using a water tank equipped with a spherical light in [12]. Attributed to their specialized equipment, their work can only applied to measurement of liquid. Mukaigawa et al. designed an inverse rendering method to acquire parameters of dipole model for more complicated shape (e.g., cube, pyramid) in [7]. For more efficient procession, they firstly recovered the diffusion reflectance and then formulate the parameter acquirement as a simple fitting problem. Whereas, Noticeable error between observation and regeneration was presented in their paper. They attribute this error to the over-approximation of dipole model. And this could also be the main motivation for us to choose directional dipole model[3] instead in our work. Parameter estimation for object with more complicated shape is also presented. Munoz et al. offered an approach to estimate scattering parameter and reconstruct 3D shape approximately from a single image without any previous knowledge (e.g., shape and light position) in [13] . One limitation of their work is the dependency on being globally convex and optically thick. Ckioulekas et al. challenged the inverse scattering problem of heterogeneous material in [14]. Their 3D shape is presented using lots of tiny cubes and scattering parameters is estimated for every cube. They made efforts to reconstruct the 3D shape of the input image.

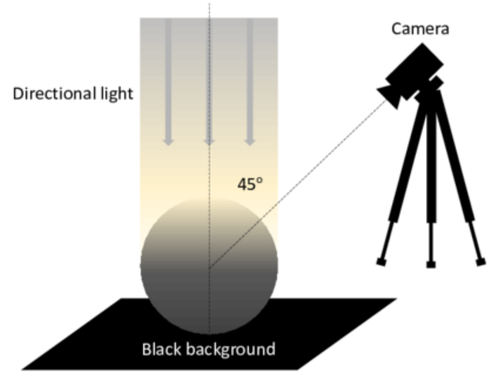

Figure 1. Measurement apparatus

C. Curvature and reflectance

Kolchin et al. presented how surface curvature affects the diffuse reflection of translucent surface by deriving the solution of diffusion equation for spherical surface in [15]. Noticeable difference between rendering results generated by their proposed solution and by dipole approximation[1] was shown in their work, indicating the effect of surface curvature on subsurface scattering. In [8], Kubo et al. developed a Curvature-Dependent Reflectance Function (CRDF) to simulate subsurface scattering more efficiently. They pointed out the fact that subsurface scattering effect tend to be more noticeable on complex surface and proposed to utilize knowledge of curvature to simulate subsurface scattering approximately. Inspired by their work, we are thus interested in inverse rendering approach considering surface curvature.

Compared with the previous work, our work focus on objects with spherical surface, which would be more complicated than the simple planar surface. While we do not provide a solution of how to utilize knowledge of surface curvature, a theoretical explanation for the observation is given.

III. Our approach

Like the general inverse rendering method, we designed a scene (figure 1) and then estimate parameters for the directional dipole model by optimizing an error function. Different from the previous work, our work focus on figuring out how the surface curvature affect the accuracy of estimation.

A. Our assumption

As mentioned previously, when ray goes into scattering material it bounces several times and then goes out of the object. According the number of bounces, scattering can

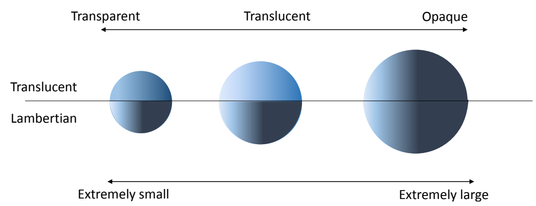

Figure 2. How surface curvature affect translucency

be literally classified into single scattering and multiple scattering. Imagining sphere with extremely high curvature (small radius) as shown in Fig. 2, we can intuitively understand that ray tends to go straight through the sphere. Singe scattering therefore dominate because of the less scattering events. In the opposite case of sphere with extremely low curvature (large radius), it is more like a Lambertian surface as scattering effect is ignorable.
According to this intuitive knowledge and theory proposed in [8], we assume that sphere with a suitable curvature can present more visible translucency effect and offer more details for estimation. In this paper, an inverse rendering method similar with [5][1] is applied for parameter estimation also from spherical surface. To validate our assumption, we test our fitting approach in simulation and estimate parameters for spheres with different curvatures on real scene respectively.

B. Image formation model

As shown in Fig. 1, sphere is placed in an ideal dark background so that no reflected light from the background. The target sphere is illuminated using a light source with given intensity and position. Instead of the usually used impulse light, a directional light is used for more details of the outgoing radiance change. Photographs are taken from 45 degree away from the surface normal direction. The outgoing radiance $L_o(x_o, \vec{\omega}_o)$ is computed by the following integral:

$$L_o(x_o, \vec{\omega}_o) = \int_A \int_{2\pi} S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i)(n_i \cdot \omega_i) d\omega_i dx_i \tag{1}$$

Where A means the surface area of sphere. With a directional light source, we can simplify equation 1 to an one-dimension integral:

$$L_o(x_o, \vec{\omega}_o) = \int_A S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i)(n_i \cdot \omega_i) dx_i \tag{2}$$

Where n_i becomes a constant vector as shown in Fig. 1. As mentioned above, single scattering becomes significant in case of high curvature. Conventionally, most BSSRDF approximation can only formulate multiple scattering and inefficiently utilize an additional ray tracer to deal with single scattering. In our work, directional dipole model is chosen for prediction. As Frisvad et al. formulate the single scattering in their BSSRDF approximation[3], we can thus cover single scattering with reasonable time consuming by applying their model. The full BSSRDF with directional dipole approximation is:

Table I
PARAMETERS FOR DIRECTIONAL DIPOLE MODEL

σ_s	absorption coefficient
σ_a	scattering coefficient
g	mean cosine o the scattering angle
η	relative index of refraction

$$S = T_{12}(S_d + S_{\delta_E})T_{21} \tag{3}$$

Where T_{12} and T_{21} is the Fresnel transmittance terms at point x_i and x_o. S_{δ_E} models single scattering that is along the refracted direction. S_d models single scattering from other direction and multiple scattering component. For explicit definition of S_d and S_{δ_E} can refer to [3]. As there is no refracted light in our scene, we simplify Eq. (3) to $S = T_{12} S_d T_{21}$. Combining Eq. (2) and Eq. (3), we get:

$$L_o(x_o, \vec{\omega}_o) = \int_A T_{12} S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) L_i(x_i, \vec{\omega}_i)(n_i \cdot \omega_i) T_{21} dx_i \tag{4}$$

With Eq. (4), we now can simulate the observation given appropriate parameters for directional dipole model.

C. Optimization problem

When we acquire a observed image of the scene, instead of rendering the whole scene, we use a 1D slice of the observed image Fig. 5)for estimation. The x-axis is the vertex angle of x_o and y-axis value of the graph can be computed with Eq. (4). To drive the directional dipole model, theoretically we need to estimate 4 unknown parameters, which are as shown in Table I As there is already established technique to measure index of refraction for objects, we decide to lay more emphasis on estimation of scattering parameters and use the reference value of different material for our estimation. By accepting the assumption of isotropic scattering in our case, we can further set $g = 0$.

Now we can estimate the remained parameter by optimizing the error function:

$$(\sigma_a, \sigma_s) = \arg\min_{\sigma_a, \sigma_s} E \tag{5}$$

Where E is the error function present the total square error between the observation and the prediction:

$$E = \sum_{i=0}^{N} (L_{obs}(p_i) - L_{pdt}(p_i))^2 \tag{6}$$

In which p_i is the position of the image pixel and N is the total number of pixels in the image. $L_{obs}(x_i)$ and $L_{pdt}(x_i)$ mean the radiance of these sample given by observation and prediction respectively. According to equation 5 and 6, we setup experiment on simulation and real scene. We will show the detail and results on section 4 and 5.

IV. EXPERIMENT ON SIMULATION SCENE

To validate our fitting approach, we firstly estimate parameters from simulation data such that affect results from real scene can be avoided. In the experiment of a real scene, we set the camera far away from the sample. Benefit from the high resolution of camera (6000 x 4000), we get a good observation of the sphere even though the camera is far away from the sphere. And the useful region in the image is only approximately 500 x 500. To reduce the procession time, we render the useful region in the image directly. As the camera is far away enough, it can be approximated to an orthographic camera to the sphere. We thus implement an orthographic camera in all of our image formation procession so that we can render image like ones in Fig. V-B easily despite the different sizes of sphere. As the input data is generated using directional dipole model, a good fitting is expected. We extract a 1D slice from the generated data as input and the estimation result is shown in Table II.

Table II
RESULTS OF DIFFERENT SPHERE IN SIMULATION

Parameter	$\sigma_a(mm-1)$	$\sigma_s(mm-1)$
Ground truth	0.0010	0.1000
Estimation	0.0010	0.1000
Ground truth	0.1000	0.0010
Estimation	0.1000	0.0010

According to the fitting result, parameters are estimated exactly from the spherical surface, which means our fitting approach is reliable.

V. EXPERIMENT ON REAL SCENE

A. Setup of environment

We test the robustness of our algorithm by estimating parameters from observed data acquired from real environment. To interpret our scene, we use a Optoma ML750 LED projector as our light source. We create the directional light by placing the projector far away enough and projecting a circle pattern to the sphere. We use a Nikon D5300 for observation. With a LED projector, background region that should be black is also illuminated as the projector cannot really project a dark light. To eliminate the noise of the stray light, we subtract a dark image, taken with projecting an black image to the scene, from each measurement and reference. To avoid reflected light form the background, we adjust the size of the projected pattern to be same as size of the spheres. In experiment on real scene, we cannot set the radiance of the light source like in simulation. Another difficulty is that the result is affected by the optical property of the camera lens and CCD, which would be unknown to us. Mathematically, we can modify equation 4 to:

$$
\begin{aligned}
&I_o(x_o,\vec{\omega}_o) = \\
&K\int_A T_{12}S_d(x_i,\vec{\omega}_i;x_o,\vec{\omega}_o)L_i(x_i,\vec{\omega}_i)(n_i\cdot\omega_i)T_{21}dx_i
\end{aligned} \tag{7}
$$

Where $I_o(x_o,\vec{\omega}_o)$ is the pixel value in point x_o viewed from direction ω_o and K is the camera sensor response. As mention above, $L_i(x_i,\vec{\omega}_i)$ and K is unknown in Eq. (8).To solve these problems, we take the reference image of a white ideal diffuser reflector(Labsphere Spectralon, $reflectance > 0.99$). According to the property of ideal diffuser (Lambertian surface), diffusive reflectance of the diffuser is $R_{diffuser} = \frac{1}{\pi}$. We place the diffuser at the same place as the sphere and illuminate it with same pattern used for the sphere. The pixel value of the diffuser is formulated as following:

$$
I_{diffuser}(x_i') = KL_o(x_i',\vec{\omega}_i')(\vec{n}_i'\cdot\vec{\omega}_i')R_{diffuser} \tag{8}
$$

In which $I_{diffuser}(x_i')$ is the pixel value of point x_i' on the diffuser. Based on the condition of directional light, intensity of radiance is a constant independent on position x_i'. Therefore we have $L_i(x_i,\vec{\omega}_i)=\overline{L}$ and $I_{diffuser}(x_i')=\overline{I}_{diffuser}$, which means that incoming radiance and observed pixel value is identical to mean of all points. We then reshape Eq. (8) to $K\overline{L}=\frac{\overline{I}_{diffuser}}{(\vec{n}_i'\cdot\omega_i')}$. In other words, $K\overline{L}$, which is needed in equation 7, can be acquired together by Eq. (8). For simplicity, we make the direction of the directional light perpendicular to the diffuser surface so that we have $\vec{n}_i'\cdot\omega_i'=1$. To configure the scene exactly, we also make some alignment to the measure tools. To ensure the direction of light source is perpendicular to diffuser surface, we project a slim ray to the mirror so that there would be a reflected ray on the lens of projector. Note that the surface of mirror is parallel to the one of diffuser. By overlapping the reflected point on the emitting point, we can make sure the direction of light source is perpendicular to the mirror, and also the diffuser. To illuminate the diffuser, we still need to replace the mirror with the diffuser. To do this, we fix the mirror and diffuser on a stage can move horizontally. Once the light direction is set, we can simply move the diffuser to the position of the mirror while preserving their parallelism. To eliminate the effect of specular reflection, we use a pair of cross-polarization to filter out the glossy surface reflection. However, doing this also bring a side effect that also eliminating the single scattering events, which will be discussed in detail in following section. To generate the scattering profile, we also need to know the x-axis, the vertex angle (θ) of point x_i when sphere is placed in the spherical coordinate system. Which can be computed by equation:

$$
\theta = \arcsin\left(\frac{y_w}{R}\right) \tag{9}
$$

where y_w is the y coordinate in the world coordinate and R is the radius of sphere. To know y_w, we can transform the

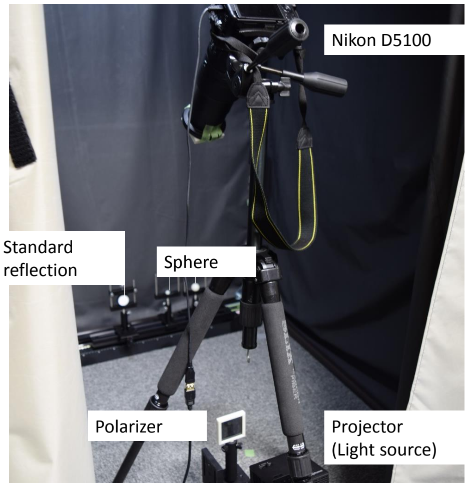

Figure 3. Environment setup

UV coordinate of a pixel to the world coordinate:

$$(x_w, y_w) = ((0.5 + u)dx, (0.5 + v)dy) \tag{10}$$

Where $dx = \frac{R}{Width_{img}}$ and $dy = \frac{R}{Height_{img}}$, which mean length in the real world for per pixel in horizontal and vertical direction. For comparison, we prepare 3 different nylon spheres as our target.

B. Result and conclusion

As described in Section 4, we take photographs for the spheres. Different from experiment on simulation scene, we do not know the ground truth of the parameters. Instead of comparing with the ground truth, we decide to compare the estimation error for different spheres. To evaluate the error of estimation for different spheres, we naturally come to error defined by equation 6. Nevertheless we found that outgoing radiance $L_o(x_o, \vec{\omega}_o)$ in smaller sphere tend to be weaker and therefore it have a lower error despite its bad fitting. That is, the sum of absolute error may not be able to equally evaluate the estimation of different spheres. For justice in evaluation, we use the following error instead:

$$E' = \frac{1}{N}\sum_{i=0}^{N} \frac{\|L_{obs}(p_i) - L_{pdt}(p_i)\|}{L_{obs_{max}}} \tag{11}$$

Fitting and parameter estimation results are shown in Fig. 5 and Table III. Rendering result using the estimated parameters is shown in Fig. V-B. Almost same level of error is observed despite of the different radius of spheres. Limited to sample available, experiment for more spheres is not done in our work. We attribute this observation to reason that the sphere is not small or big enough to lose the translucent effect.

We also make the comparison with dipole in our experiment on real scene (Fig. 5, Table III). Estimation error of dipole model grows acutely in case that $R = 1.0$mm.

Table III
RESULTS OF DIFFERENT SPHERE IN REAL SCENE

Directional dipole model			
Radius(mm)	6.36	9.55	12.72
σ_a	0.0000	0.0046	0.0025
σ_s	0.3600	0.4428	0.3549
E'	0.1421	0.1338	0.1857
Dipole model			
Radius(mm)	6.36	9.55	12.72
σ_a	0.0390	0.0417	0.0625
σ_s	0.0551	0.0861	0.0801
E'	0.5014	0.2013	0.2904

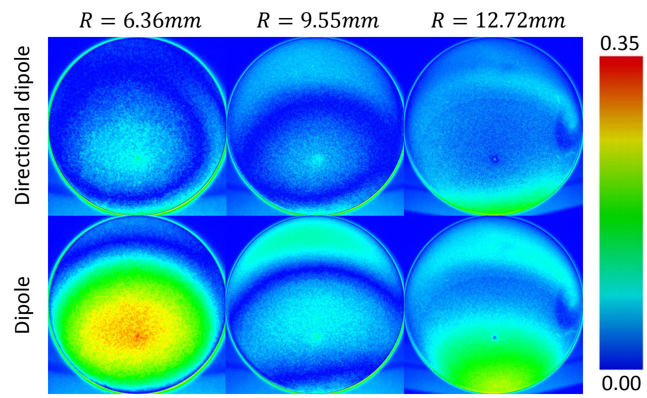

Figure 4. Reproduction compared with real observation

And directional dipole model also presents more stable result than dipole do. However, it is still show a better estimation at the middle range of the curvature, which also meet our expectation. In a whole estimation by directional dipole model have a lower error. Given all of these experiment results, we validate our assumption that curvature of the sphere affect the estimation results and verify the good performance of directional dipole model in parameter estimation. We can make a further assumption that accuracy of the conventional inverse rendering method for parameter estimation can be improved by well utilizing the preliminary of surface curvature.

VI. Discussion

A. Specular reflection

Originally, we select directional dipole model for our estimation because of its capability of covering single scattering. Nevertheless, we use a pair of cross-polarization to filter out specular reflection, which meanwhile eliminate the single scattering events. Despite this, we still have better result in directional dipole model than the original dipole model. We attribute this to the possibility that directional dipole model can also model multiple scattering better than dipole.

B. Assumption of Directional Dipole Model

Same as dipole model, directional dipole model solve the diffuse function on semi-infinite planar surface. That is, strictly, directional dipole is theoretically correct only on

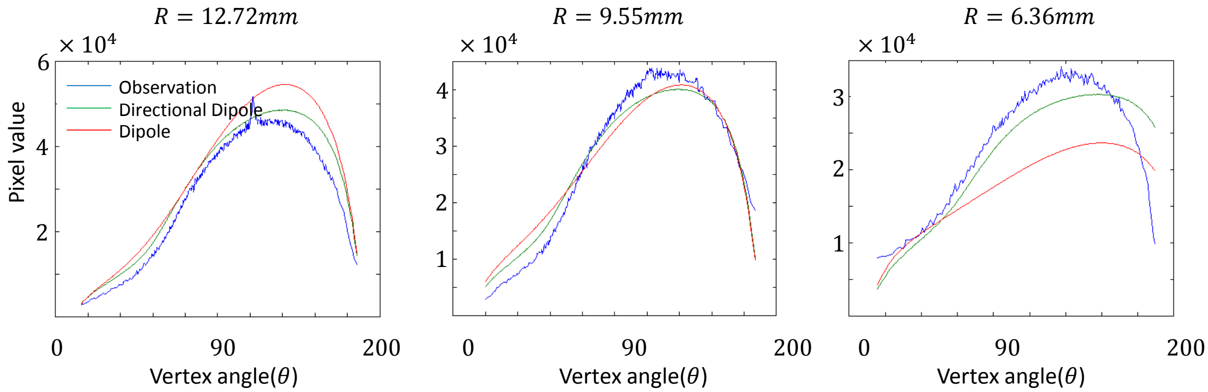

Figure 5. results of real scene

semi-infinite planar surface. When directional dipole model is used for non-planar surface, it become inaccurate, though, it still have a better performance than dipole model. Referring to the results shown in [3], we believe that directional dipole model is most robust to surface shape by now.

C. *Preliminary of shape*

Similarly to most previous inverse rendering method, one limitation of our method is the dependency on given shape. Because of the ambiguity between the shape and subsurface scattering parameters, simultaneous estimation of shape and parameters would be a challenging task. Iterative approach like work in [4] may be a interesting direction for solving this problem.

D. *Slow procession*

In our work, it takes hours to finish one estimation. We attribute this low efficiency to the high computational cost of the path tracing. For each x_o in Eq. (4), we run a Monte Carlo approach with uniform sampling to compute the integral over the surface. We can of course implement more effective sampling approach (e.g. importance sampling) to solve the problem, though, we choose not to do this in our work.

VII. Conclusion and Future Work

In this paper, we propose an inverse rendering method for estimating scattering parameters from spherical surface. Different from the previous work, we pay attention to a factor, surface curvature, which is generally ignored. We make an assumption that surface curvature affects the estimation result and design experiment on simulation and real scene to validate our approach. To better do the parameter estimation on spherical surface, we also choose the directional dipole model newly introduced in [3] as our prediction model. Experiment results show that as surface curvature changes from extremely low to extremely high, there exist a range of curvature that minimize the estimation error. We can thus further say that we can improve the estimation quality by choosing a most suitable curvature for estimation. By comparing the estimation result of directional dipole model and dipole model, we also validate our choice of directional dipole model. In future work, we plan to apply our assumption to estimation of surface with changing curvature, instead of surface with constant curvature like sphere. We expect to achieve a better estimation by choose the suitable data in the image according to the surface curvature.

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