

No.7

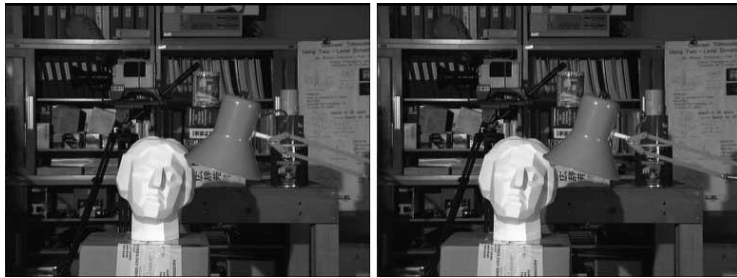
陰影解析

Shape from intensity

担当教員：向川康博・田中賢一郎

# Shape-from-X

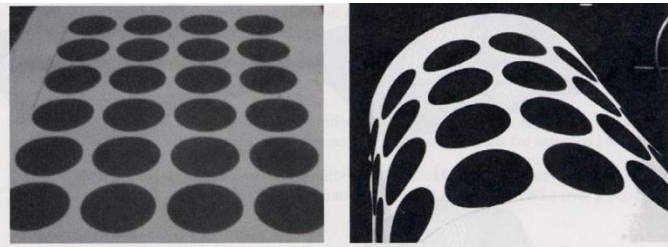
- Shape estimation from "X" as a clue



Stereo / Motion



Triangulation



Texture



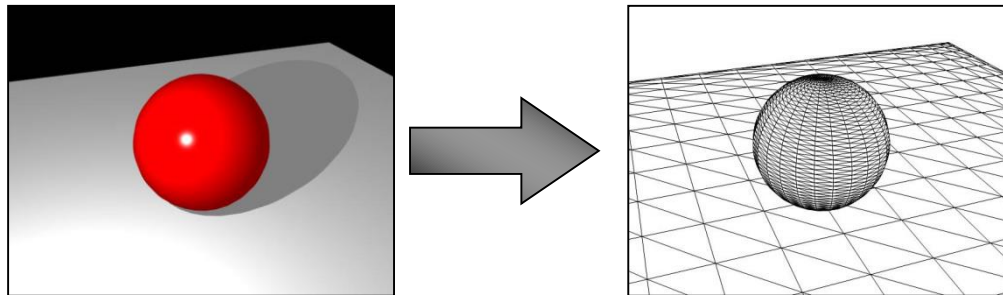
Focus / Defocus



Time

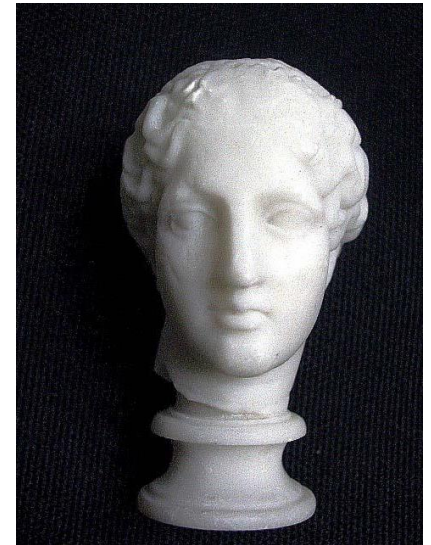
# Shape-from-Intensity

- Shape estimation with observed intensity as a clue
- Inverse process of rendering to determine pixel intensity
  - Inverse geometry
  - **Known** : Illuminations and reflectance properties
  - **Unknown** : Scene shape
- Clue
  - Shading, specular reflection, shadow, etc.



# Shape-from-Shading

- Shape estimation based on shading
- Photometric information contains a lot of clues about geometry.
  - Surface normal
  - Depth
- Start with a simple problem setting
  - A point light source at infinity (parallel light)
  - Fixed camera and target object
  - Perfect Lambert reflection  
(No specular reflection, No shadow)
  - No global illumination

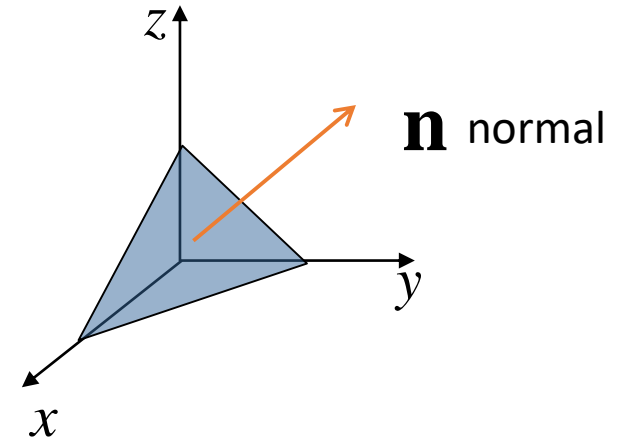


# Surface normal

- Plane in 3D Euclidean space

$$Ax + By + Cz + D = 0$$

or 
$$\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$$



- Partial differential of  $z$  with  $x$  and  $y$  to define the inclination.

$$-\frac{\partial z}{\partial x} = \frac{A}{C} = p \quad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$$

- Unit normal vector from the inclination.

- defined by two parameters

$$\mathbf{n} = \frac{(p, q, 1)^T}{\sqrt{p^2 + q^2 + 1}}$$

# Illumination and reflection

## ■ Parallel illumination

- ▣ Ideal illumination coming from a point light source at infinity
- ▣ Every surface point is illuminated with the same irradiance from the same direction
- ▣ Unit parallel illumination vector

$$\mathbf{s} = \frac{(p_s, q_s, 1)^T}{\sqrt{p_s^2 + q_s^2 + 1}}$$

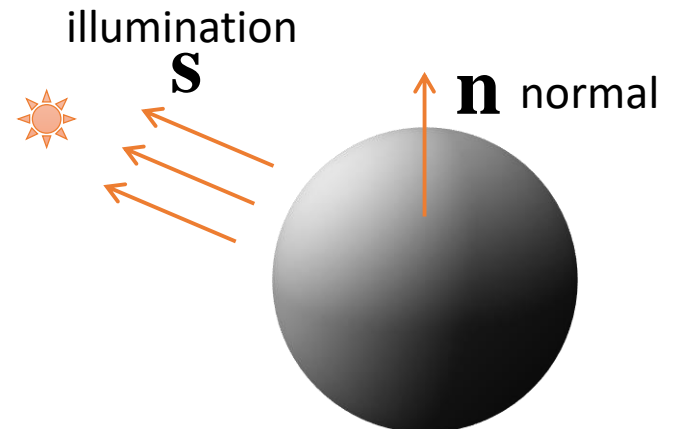
## ■ Lambert diffuse reflection

- ▣ Ideal reflection that uniformly reflects incident light in all directions

▣  $i$  : Observed intensity

▣  $\rho$  : Lambert diffuse reflectance ( $\rho \geq 0$ )

$$i = \rho \mathbf{s}^T \mathbf{n}$$



# Problem setting

■ Known: illumination and observed intensities

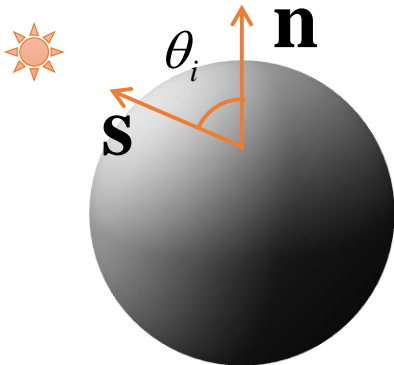
■ Unknown: normal

$$\text{Unknown } \mathbf{n} = \frac{(p, q, 1)^T}{\sqrt{p^2 + q^2 + 1}} \quad \mathbf{s} = \frac{(p_s, q_s, 1)^T}{\sqrt{p_s^2 + q_s^2 + 1}} \quad \text{Known}$$

■ For simplicity, assuming that diffuse reflectance  $\rho = 1$

$$I = \cos \theta_i = \mathbf{n}^T \mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}} = R(p, q)$$

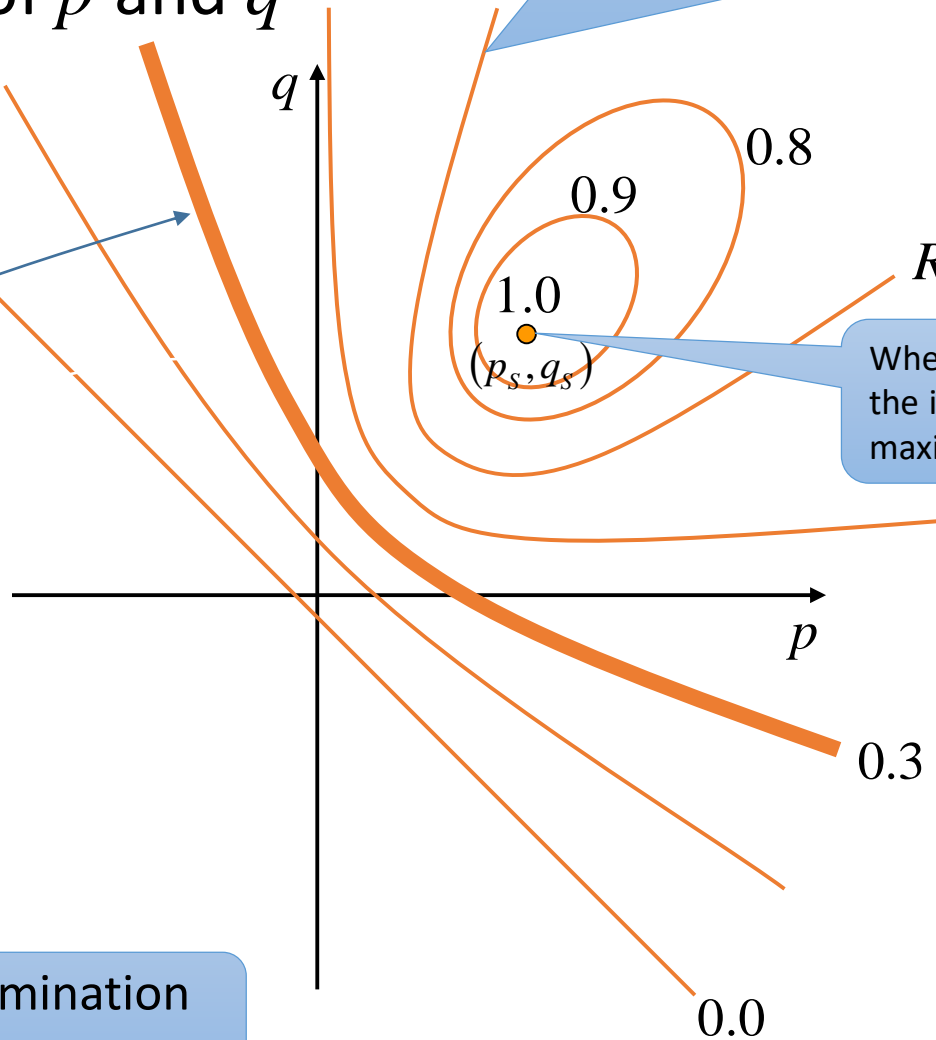
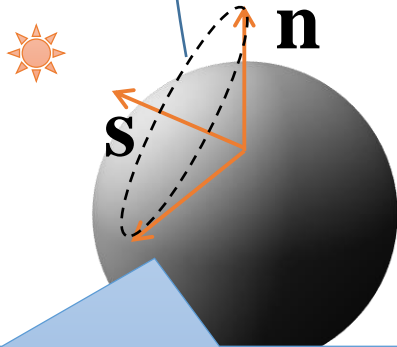
Reflectance map



# Reflectance map

■  $R$  is a function of  $p$  and  $q$

When intensity  $i$  is observed, the normal is limited on a curve within  $(p, q)$  solution space.



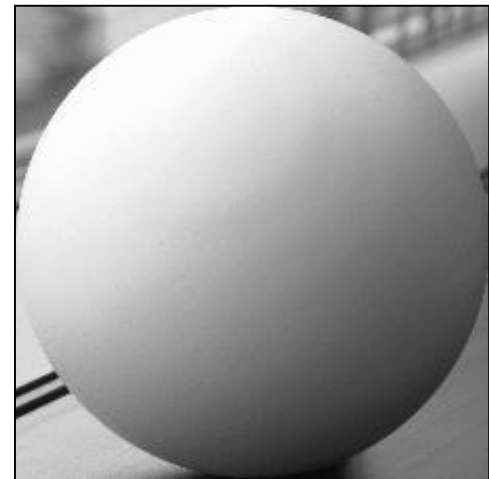
When  $(p, q) = (p_s, q_s)$  the intensity has the maximum.

Only the angle between illumination direction becomes known.



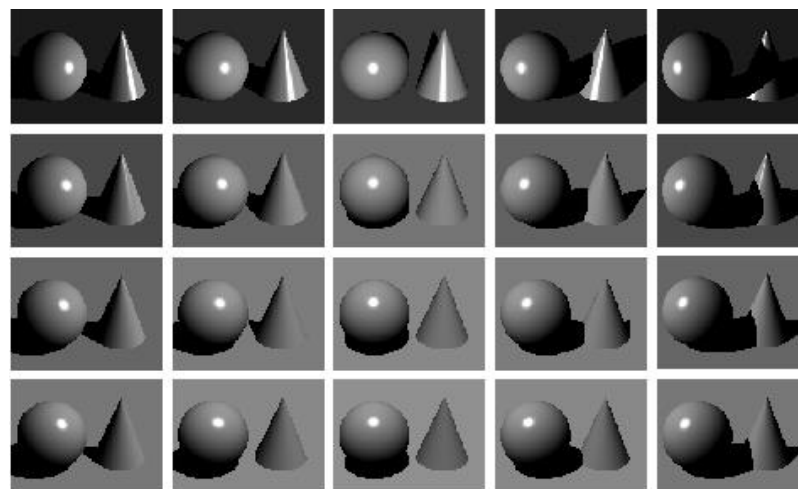
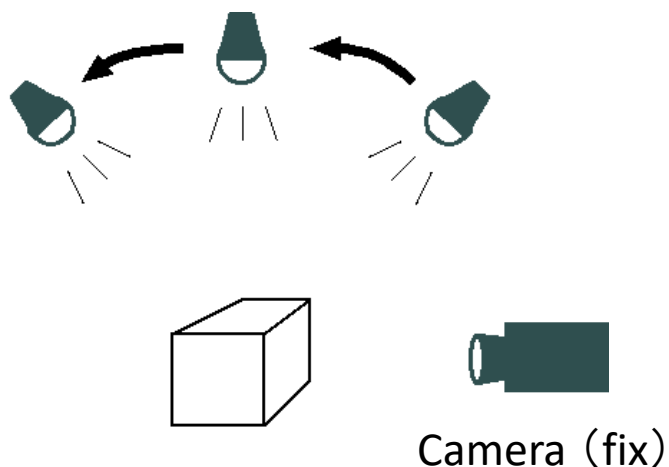
# Shape-from-Shading

- The normal  $(p, q)$  cannot be uniquely determined from one observed intensity
  - One equation and two unknown parameters
- Need to add some information
  - Assumption that the object surface is smooth
  - Prior knowledge of shape
  - Increase Illumination directions



# Photometric Stereo (照度差ステレオ)

- Taking multiple images with changing illumination directions
- Solving ambiguity in the reflectance map
- Assuming Lambert diffuse reflection, it can be solved linearly



# Example of Photometric Stereo



Input images



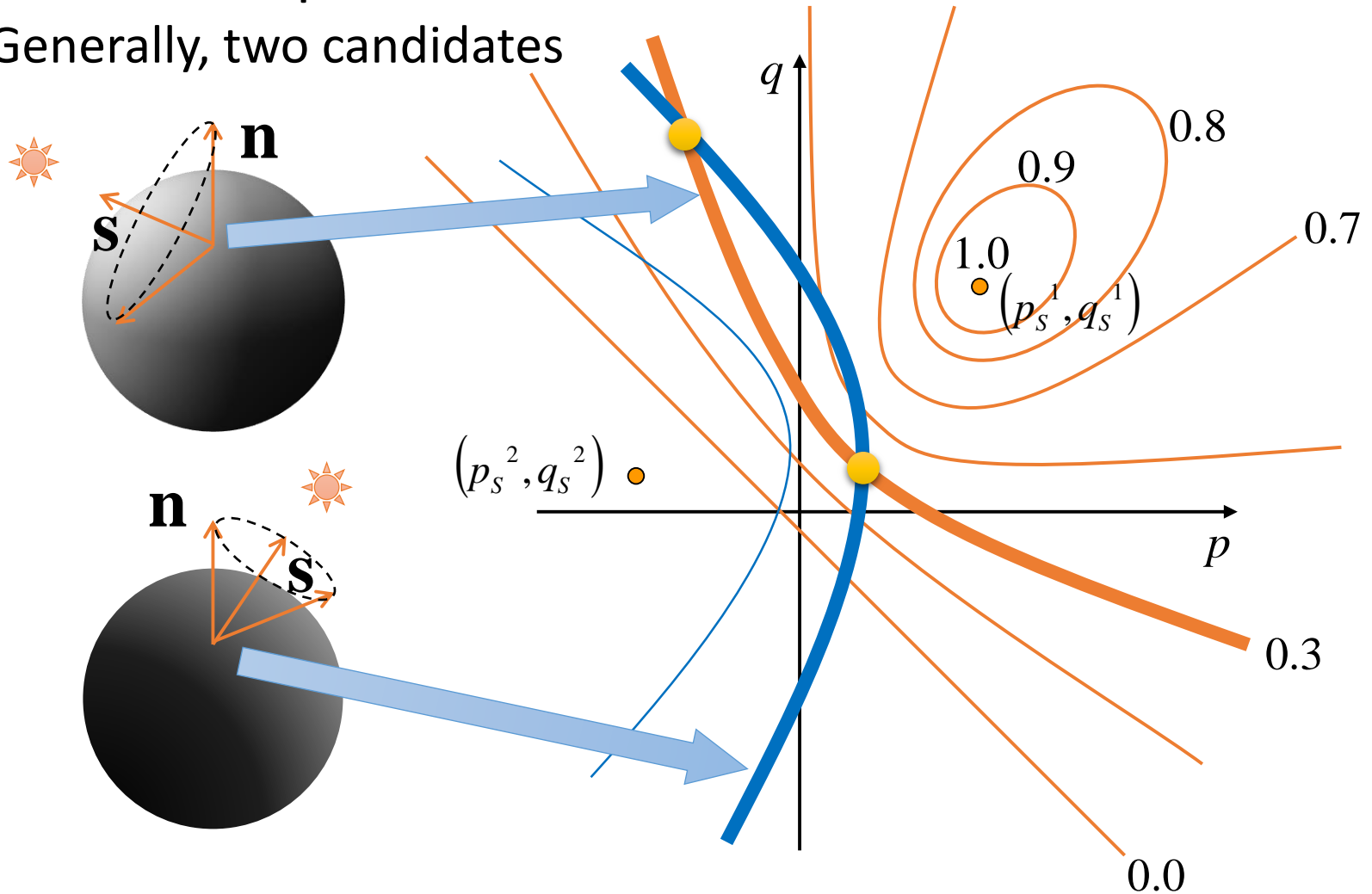
Normal map



# In the case of two light sources

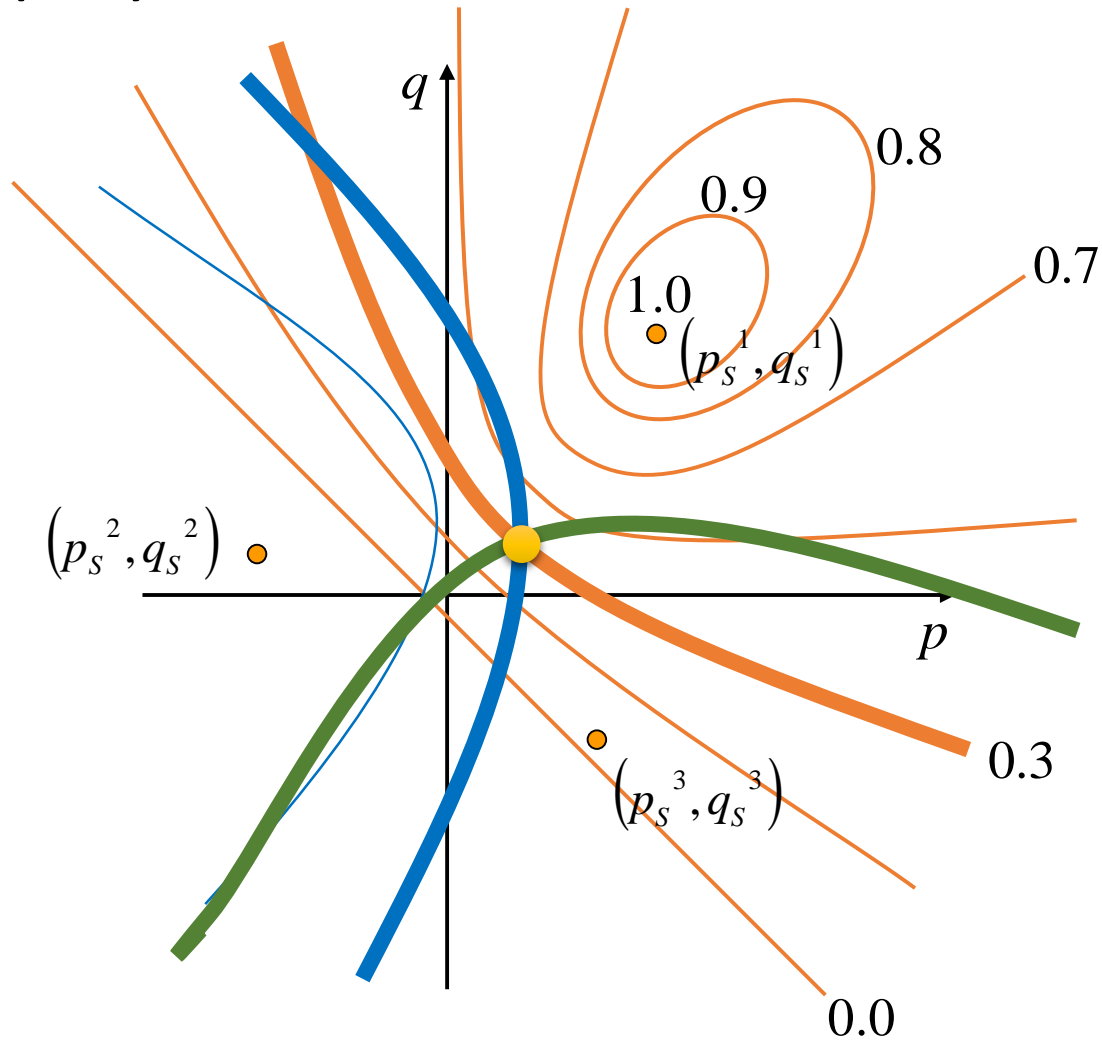
- The intersection of the two solution curves becomes new solution space

- Generally, two candidates



# In the case of three light sources

- Solve uniquely

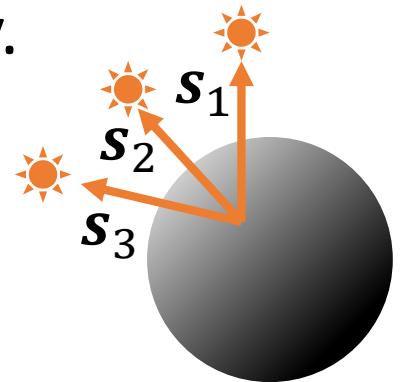


# Linear solution in the case of three light sources

- Assuming that three observation intensities ( $i_1, i_2, i_3$ ) were obtained for a pixel under three different illumination directions ( $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3$ )

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \rho \mathbf{n} \xrightarrow{\text{Express by matrix}} \mathbf{i} = \mathbf{S} \tilde{\mathbf{n}} \xrightarrow{\text{Inverse matrix}} \tilde{\mathbf{n}} = \mathbf{S}^{-1} \mathbf{i}$$
$$\begin{cases} \rho = \|\tilde{\mathbf{n}}\| \\ \mathbf{n} = \tilde{\mathbf{n}} / \|\tilde{\mathbf{n}}\| \end{cases}$$

- Stable because it can be solved linearly.
- Reflectance ( $\rho$ ) is also estimated at the same time.



# In the case of more than three light sources

- Assumed that observed brightness  $(i_1, i_2, \dots, i_M)$  was obtained under  $M (> 3)$  different illumination directions  $(\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_M)$

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_M \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \vdots \\ \mathbf{s}_M^T \end{bmatrix} \rho \mathbf{n} \quad \xrightarrow{\text{Express by matrix}} \quad \mathbf{i} = \mathbf{S} \tilde{\mathbf{n}}$$

- Since the illumination matrix  $\mathbf{S}$  is not a square matrix, calculated using a pseudo inverse matrix

$$\tilde{\mathbf{n}} = (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{i} = \mathbf{S}^+ \mathbf{i}$$

Moore Penrose(ムーア・ペンローズ)inverse matrix

- Least squares method assuming that observation error is Gaussian distribution
  - more stable and accurate solution

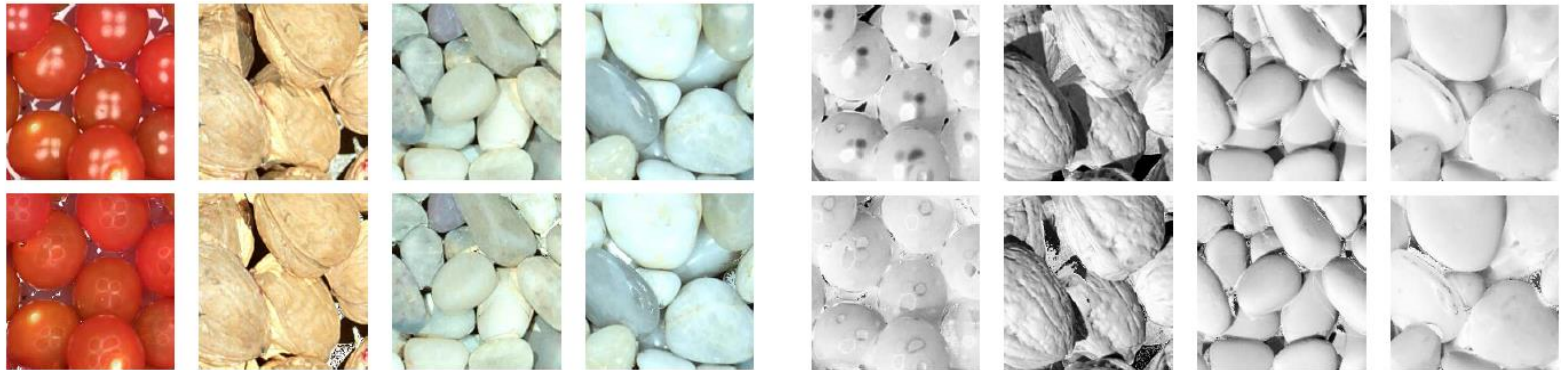
# The merit of multiple light sources

- Avoid specular reflection and shadow
  - Assuming that pure Lambert diffuse reflection can be observed in at least three images.



Input image

without



with

Consideration of  
specular reflection  
and shadow

Estimated reflectance

Shading generated from  
the estimated normal

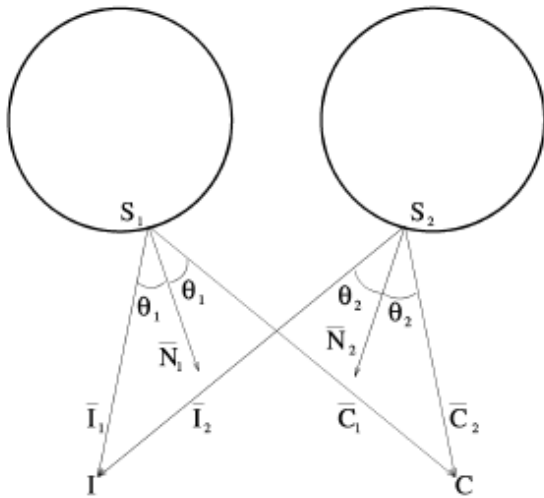
Barsky, S, et al, The 4-source photometric stereo technique for three-dimensional surfaces in the presence of highlights and shadows



# Photometric Stereo in Parthenon



- Near light source photometric stereo.
- Two black hemispheres to determine light source positions.



[Per Einarsson et al., Photometric Stereo for Archeological Inscriptions, 2004]

# Summary of the number of light sources

## ■ 1 light source

- Shape-from-Shading

## ■ 2 light sources

- Photometric Stereo (照度差ステレオ法)

## ■ 3 light sources

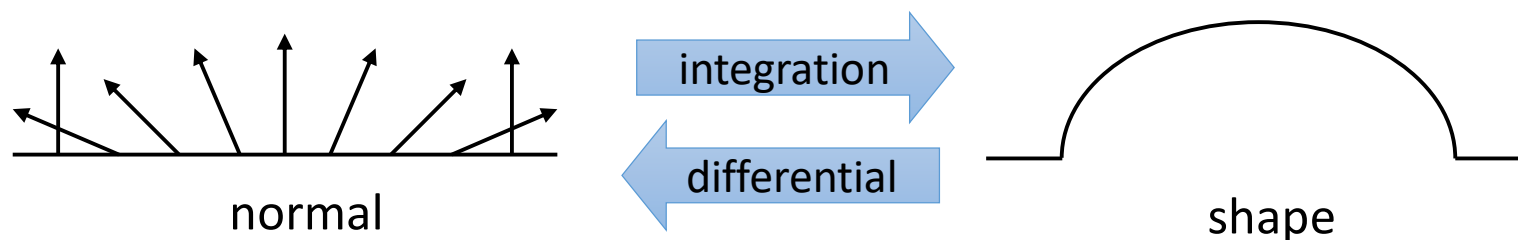
- Uniquely solved Photometric Stereo
- Simultaneous estimation of reflectance and normal

## ■ More light sources

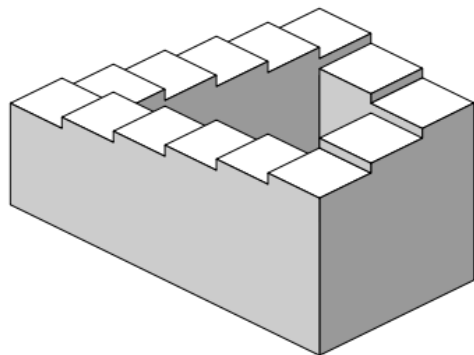
- Robust to specular reflection and shadow

# 3D shape and normal

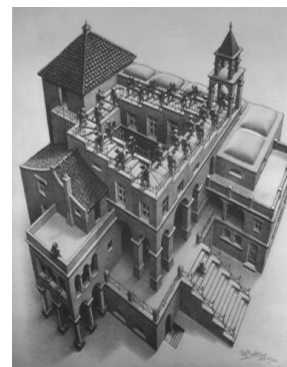
- Even if normals are known, 3D shape cannot be uniquely determined.
  - ▣ Height ambiguity due to different integral path.
  - ▣ Differential is easy, but integration is difficult.



- Ambiguity in height



A "Penrose stairs" optical illusion

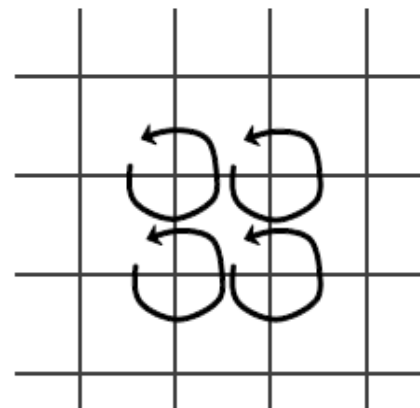
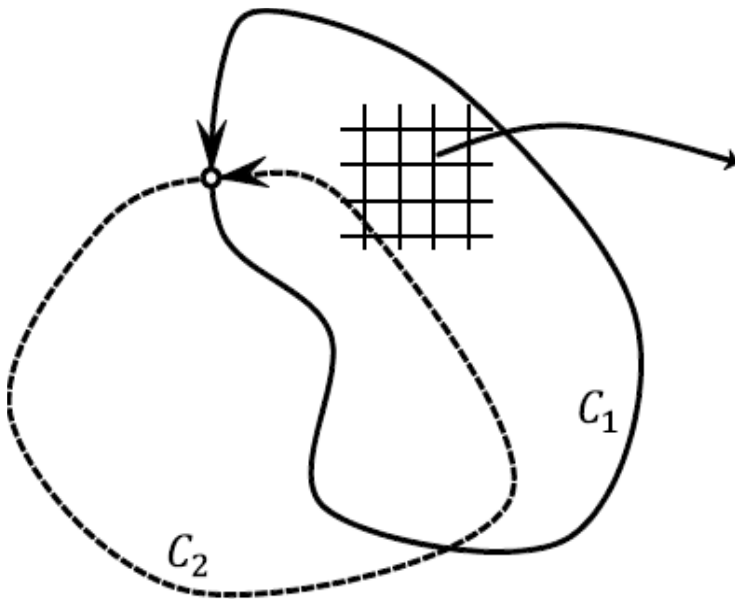


"Ascending and Descending"

# Integrability (可積分性)

- When integrated along a closed loop on a smooth surface, the integral value becomes zero.
  - ▣ Independent to the integral path.
  - ▣ Minimization of objective function  $E(Z)$

$$E(Z) = \sum_{i,j} \left[ \left( \frac{\partial Z}{\partial x} + p \right)^2 + \left( \frac{\partial Z}{\partial y} + q \right)^2 \right]$$

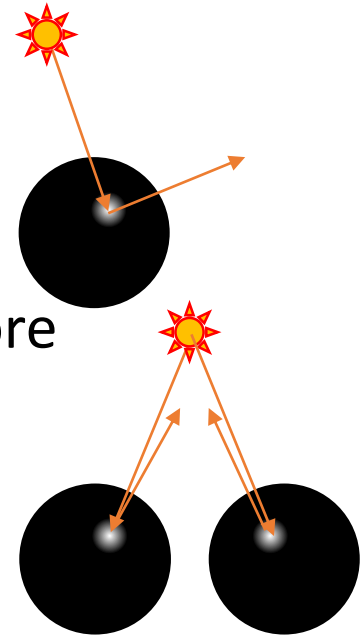


$$\left( \frac{\partial Z}{\partial x}, \frac{\partial Z}{\partial y} \right) = (-p, -q)$$

$$\text{curl}(p, q) = 0$$

# Estimation of light source direction/position

- Use reference object with known shape
  - ▣ Estimate light source direction by a black sphere
  - ▣ Estimate near light source position by two or more black spheres
  - ▣ In the outdoor, high dynamic range measurement is required
    - Mirror sphere for ambient light
    - Black sphere for position of sun
    - Diffuse sphere for brightness of ambient light



# When the illumination direction is unknown

- Illumination directions and strengths are unknown:
  - Uncalibrated Photometric Stereo (未校正照度差ステレオ)

$$\begin{bmatrix} i_{11} & \cdots & i_{1N} \\ \vdots & \ddots & \vdots \\ i_{M1} & \cdots & i_{MN} \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_M^T \end{bmatrix} [\rho_1 \mathbf{n}_1 \quad \cdots \quad \rho_N \mathbf{n}_N]$$

- Singular value decomposition  $\mathbf{I} = \mathbf{S}\tilde{\mathbf{N}} = \mathbf{U}\Sigma\mathbf{V}^T$   
(特異値分解)

$$\begin{cases} \mathbf{S} = \mathbf{U}'(\Sigma')^{\frac{1}{2}} \\ \tilde{\mathbf{N}} = (\Sigma')^{\frac{1}{2}}\mathbf{V}^T \end{cases}$$

The diagram illustrates the SVD decomposition of matrix  $\mathbf{I}$  (size  $[M \times N]$ ) into matrices  $\mathbf{U}$  (size  $[M \times M]$ ),  $\Sigma$  (size  $[N \times M]$ ), and  $\mathbf{V}^T$  (size  $[N \times N]$ ). A dashed red box highlights a sub-matrix of  $\mathbf{U}$  and  $\Sigma$  used for rank-3 approximation.

# Bas-Relief Ambiguity (浅浮き彫りの曖昧性)

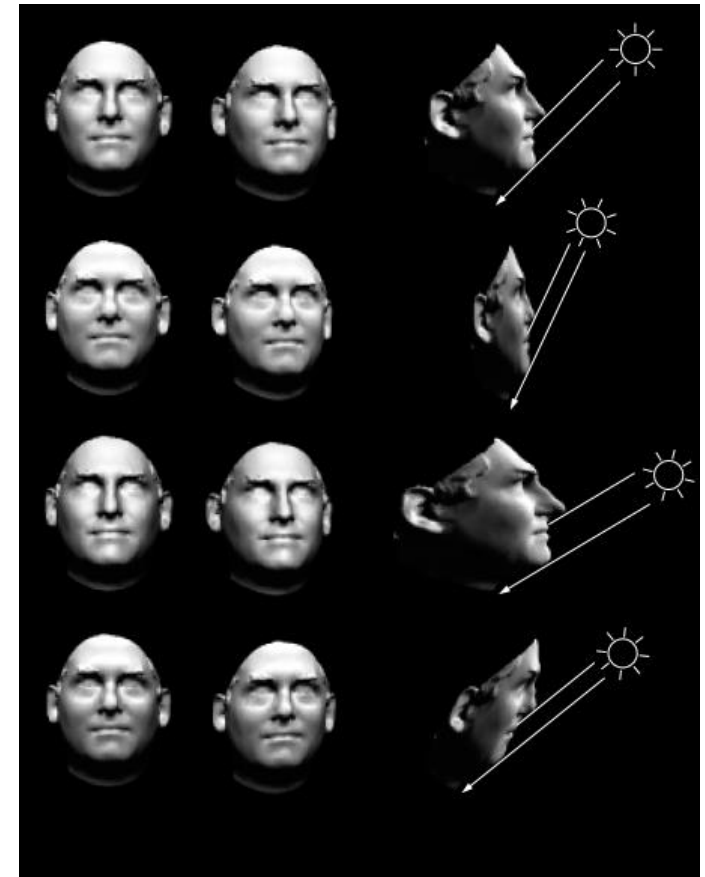
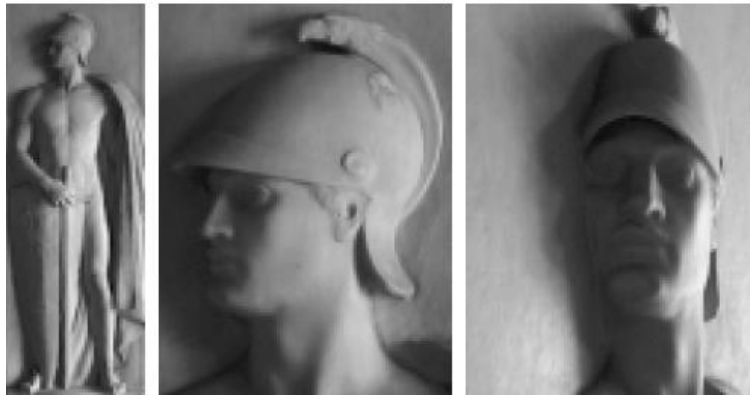
- Decomposition of a matrix is not uniquely determined.

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{N}} = (\mathbf{S}'\mathbf{H})(\mathbf{H}^{-1}\tilde{\mathbf{N}}')$$

$\mathbf{H}$  is any  $3 \times 3$  matrix

- Linear uncertainty

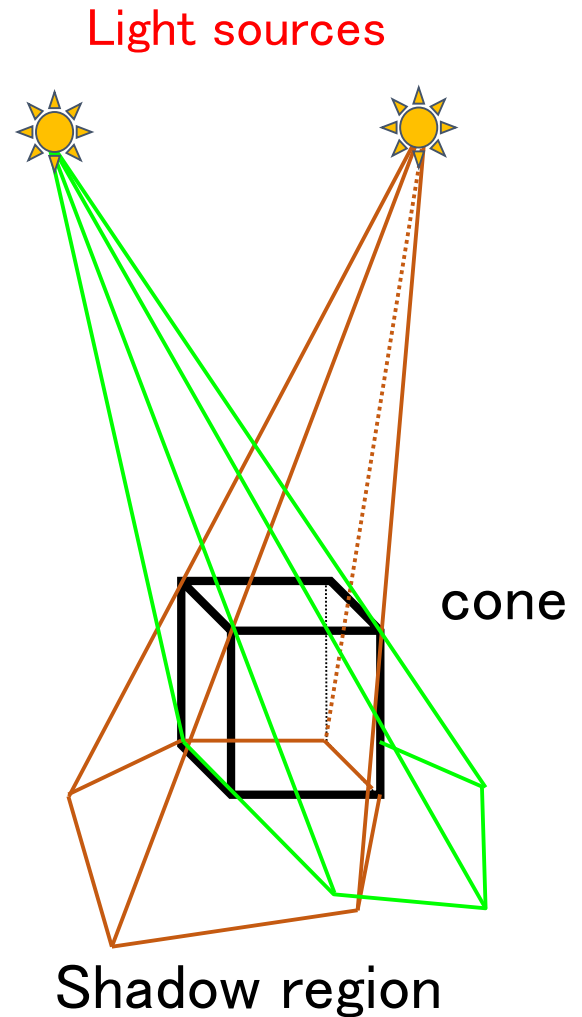
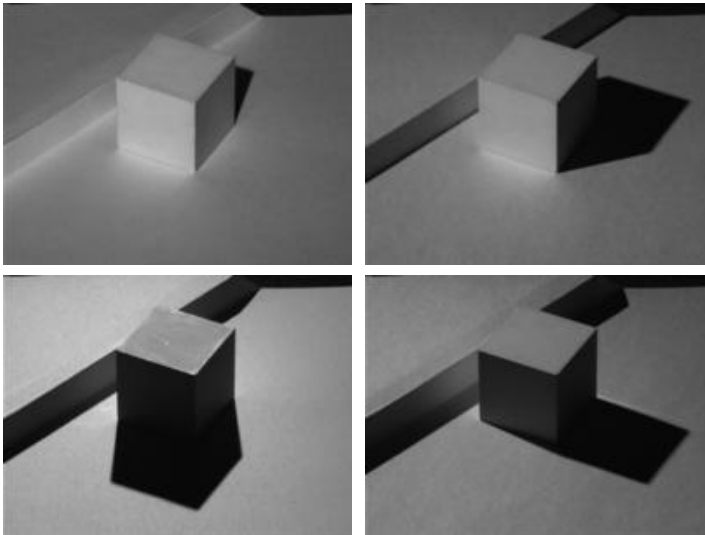
- ▣ Different incident light and normal pairs produce the same shading



# Shape-from-Shadow

- Limited existing space in the cone
  - top vertex: light source
  - bottom surface: shadow region
- Logical AND of many cones
- Estimate rough convex hull shape

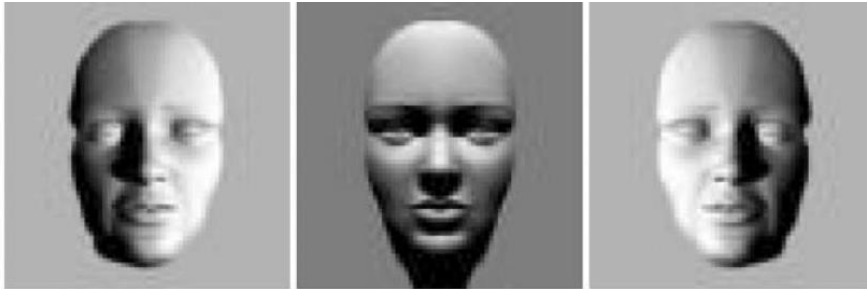
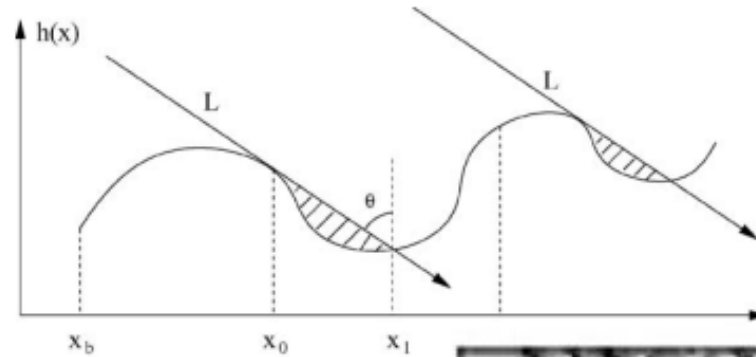
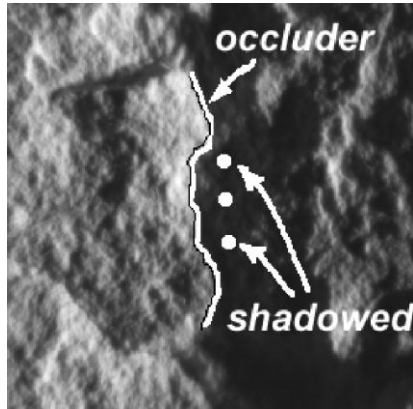
(凸包形状)





# Shadow graphs

- Restrict the existing space of objects from shadows



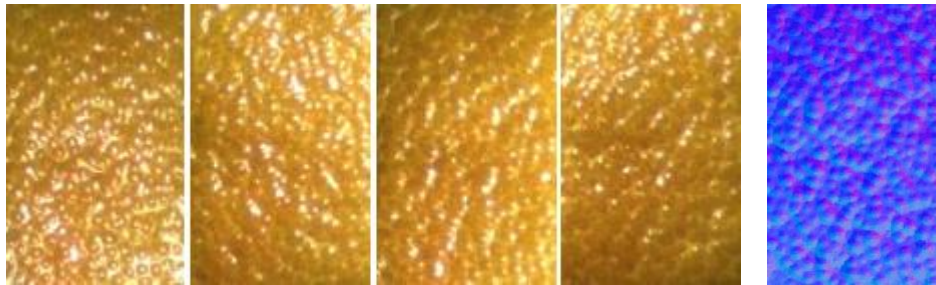
A part of the 48 input images



Shape estimation

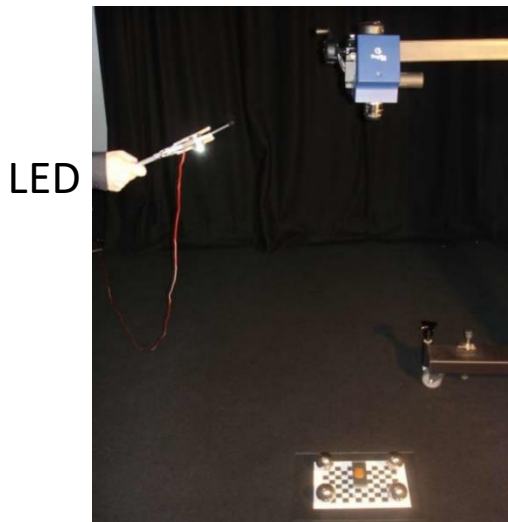
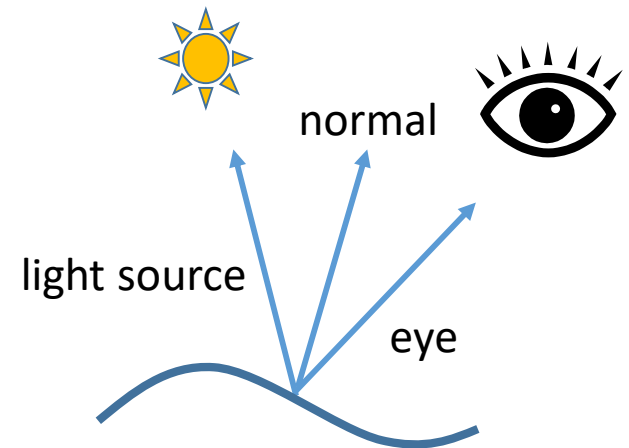
# Shape-from-Specularity

- Shape estimation based on specular reflection
  - Strong specular reflections around the mirror direction



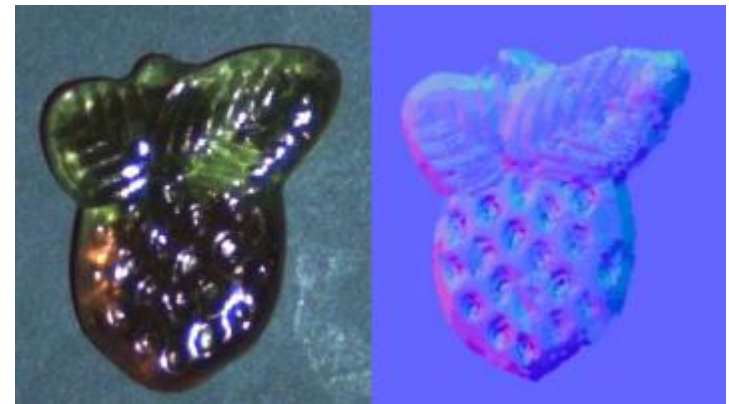
A part of 65 input images

Estimated normal map



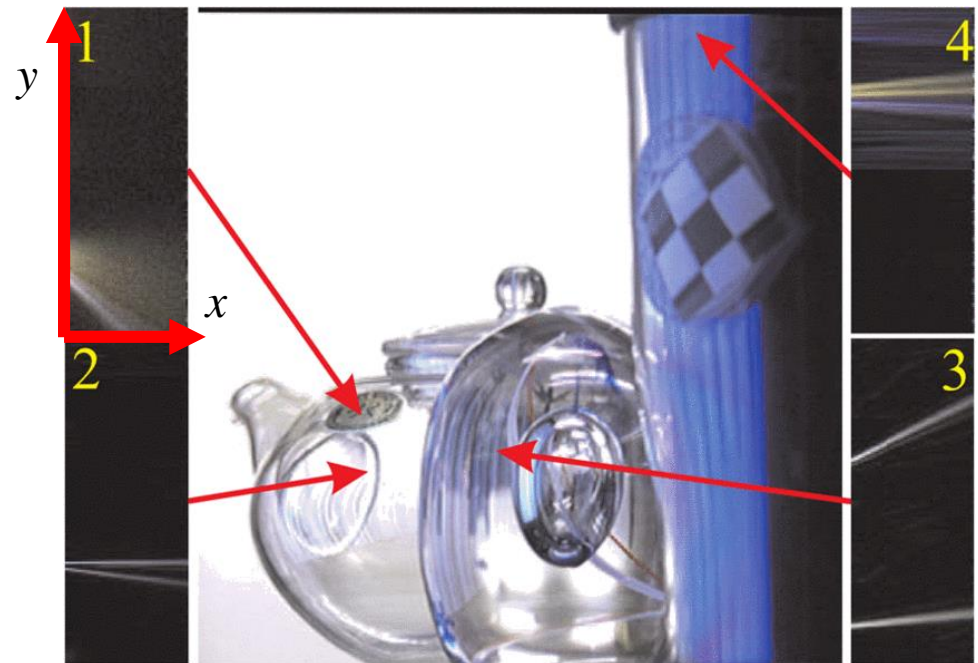
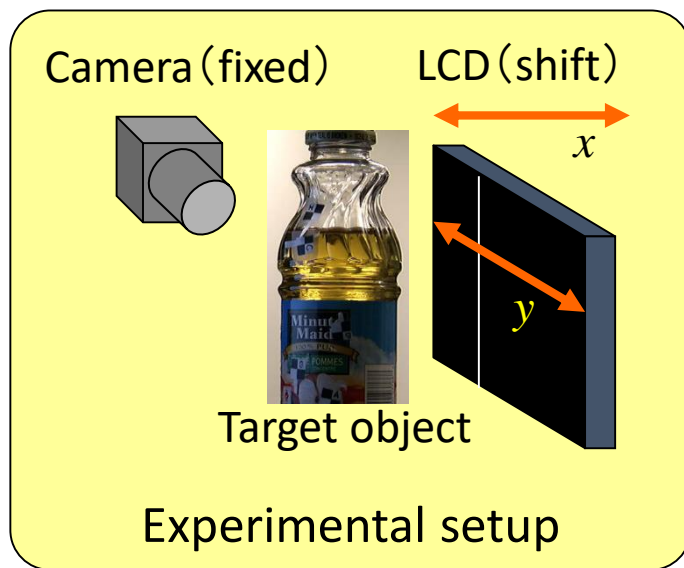
Camera  
(12bit, 1300x1030)

Metallic ball for estimating  
light source position



# Scatter-Trace Photography

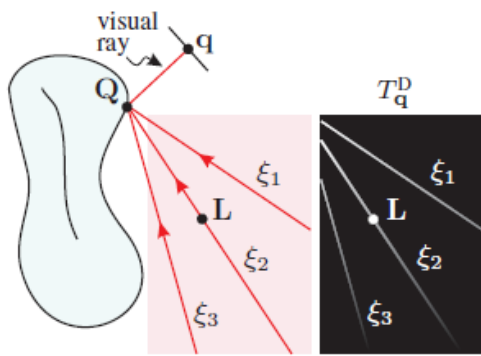
- Shape estimation based on specular reflection
- Complex transparent scene with inhomogeneous interior
  - Observed reference pattern on the transparent object
  - Shape estimation by hypothesis and verification



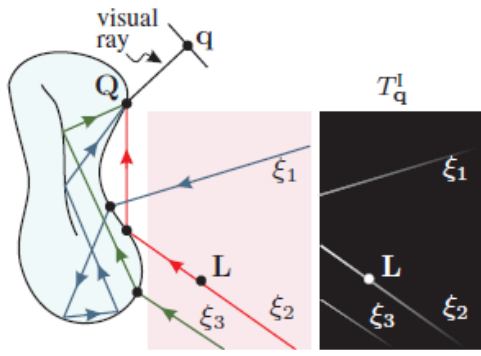
Example of **Scatter-Trace** at each point on the surface

# Analysis of Scatter-Trace

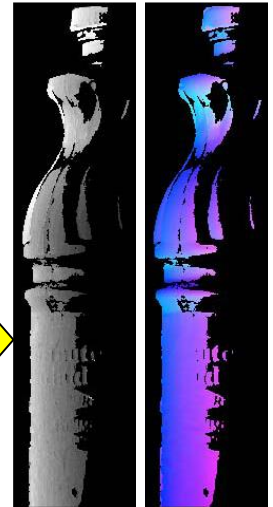
- If direct reflection on the surface, Scatter-Trace stripes
  - intersect with the eye and the surface of the object
  - monotonically decrease as the distance increases



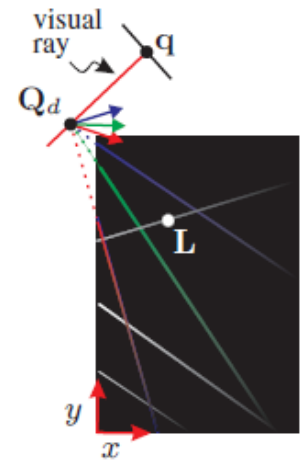
Scatter-Trace of direct reflection



Scatter-Trace of global components

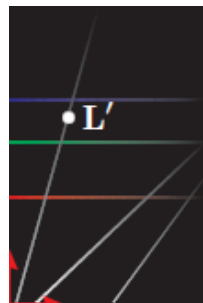


depth normal



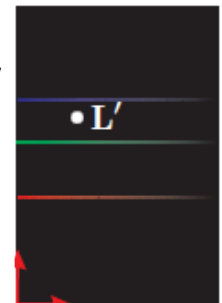
**Hypothesis**

Assuming a depth, coordinate transformation at the distance from the intersection.



**Verification**

Removing non-monotonically decreasing component → If the direct reflection components remain, the assumed depth is correct.



# Example-based Photometric Stereo

[Aaron Hertzmann and Steven M Seitz, 2005]

## ■ How to handle any BRDF?

- ▣ Reference object whose shape is known and BRDF is same with the target object to be measured.
- ▣ Used as a look up table to find similar reflection properties.



Examples of the input images



Estimated normal

- ▣ If reference objects can be prepared, any BRDF can be treated.



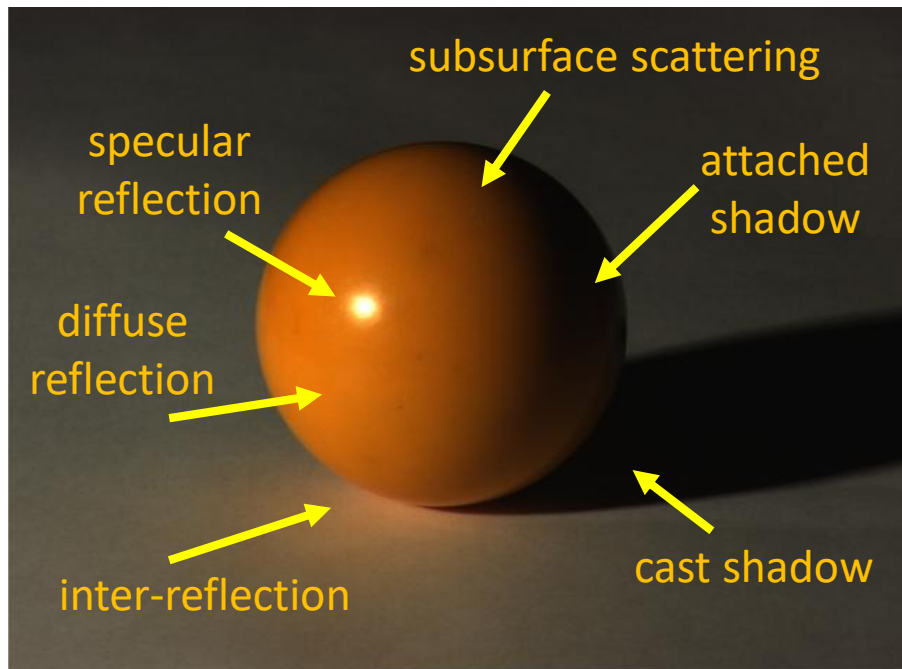
# Separation of reflection components (反射成分の分解)



# Real scene

- Many phenomena mix in real image.
- Use complex model or decompose in advance.

$$\mathbf{i} = V(\underbrace{\max(\mathbf{S}^T \tilde{\mathbf{n}}, 0)}_{\text{cast shadow}}) + \underbrace{S}_{\text{Specular reflection}} + \underbrace{G}_{\text{Global components (inter-reflection, subsurface scattering,...)}}$$



# Dichromatic reflection model (Shafer 1985)

(2色性反射モデル)

■ Reflected light = **Diffuse reflection** + **Specular reflection**

■ Diffuse reflection (拡散反射):

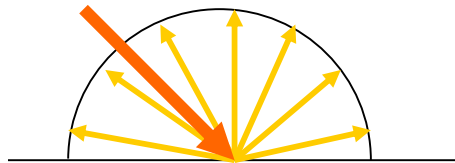
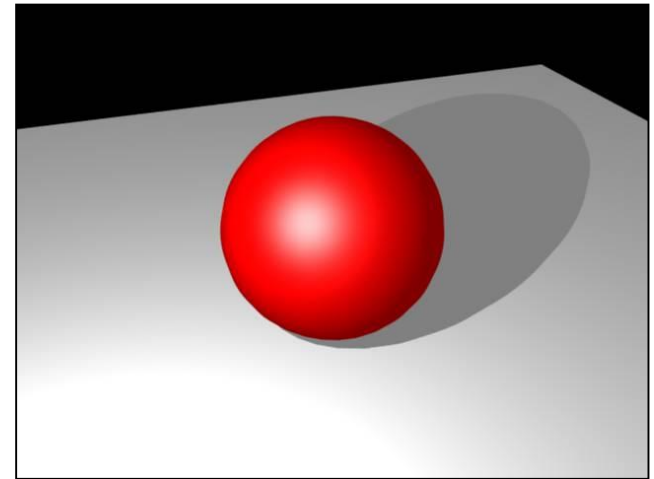
□ Reflection inside the surface layer

□ Object color

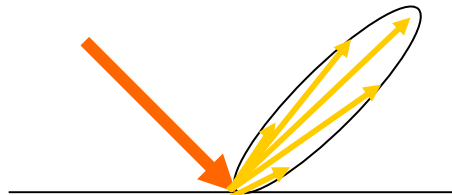
■ Specular reflection (鏡面反射):

□ Reflection at the border between air and surface layer

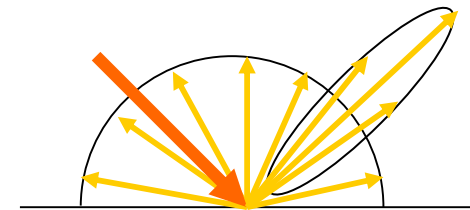
□ Light color



Diffuse reflection



Specular reflection

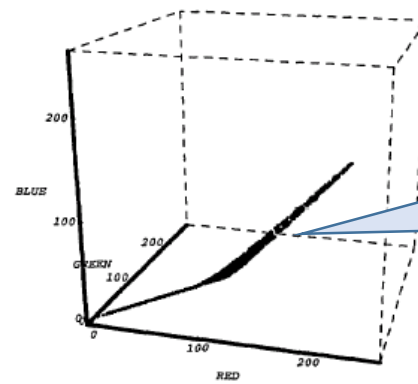
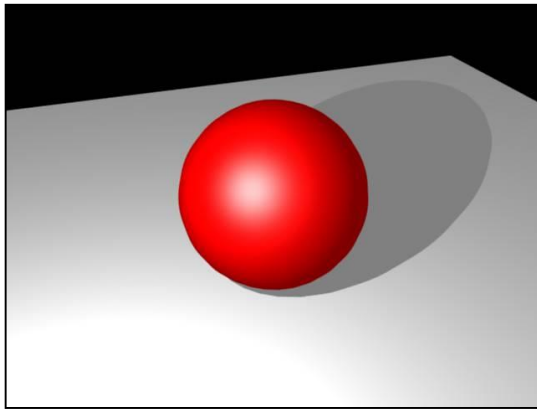


Sum of both reflection

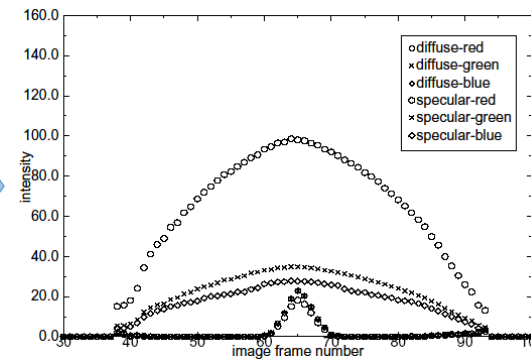
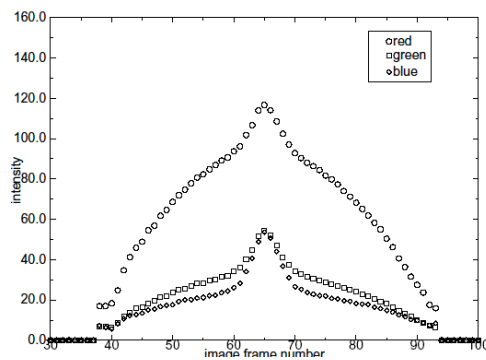


# Dichromatic reflection model

## ■ Decomposition based of color difference

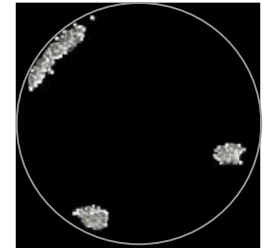


Object color  
+  
Illumination color



# Viewpoint dependency

- Diffuse reflection : independent on viewpoint
- Specular reflection : dependent on viewpoint
  - Assume uniform specular reflection characteristics
  - Simultaneous estimation of light source distribution from specular reflection components

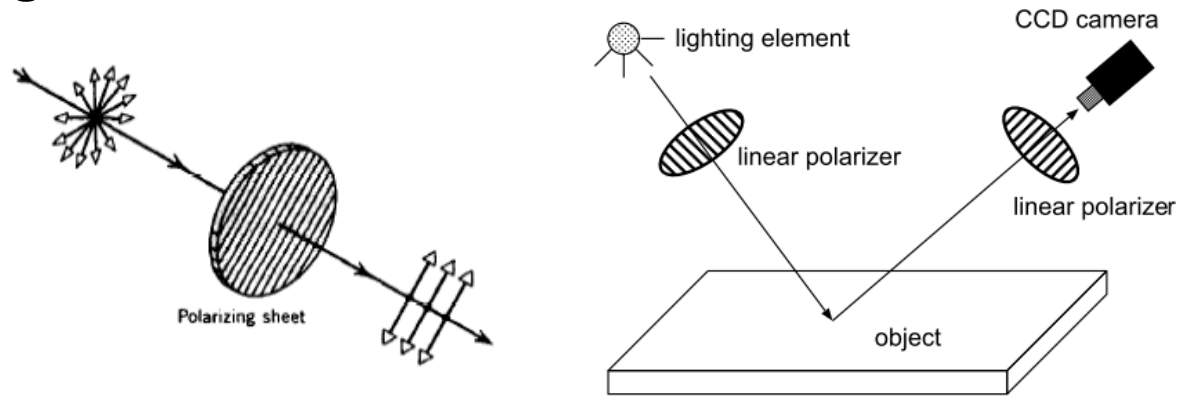


Light source  
distribution

Input image (different viewpoint ) Diffuse reflection Specular reflection

# Difference in polarization (偏光)

- Separate specular reflection components using polarization
- Simultaneous estimation of specular reflection parameter and near light source position
- Assuming monochromatic uniform reflection characteristics



Input image

Diffuse reflection component

Specular reflection component

[ 原02 ]

# RANSAC-based method



- Convert real image into pure Lambert image based on RANSAC (RANDOM SAMPLE CONSENSUS)
  - ▣ Choose three images randomly from the input image set, converted to fully satisfy Lambert model (linearization)
  - ▣ Remove shadows and specular reflections as outliers

$$\mathbf{i} = V(\max(\mathbf{S}^T \tilde{\mathbf{n}}, 0)) + S$$

attached shadow  
cast shadow                      Specular reflection

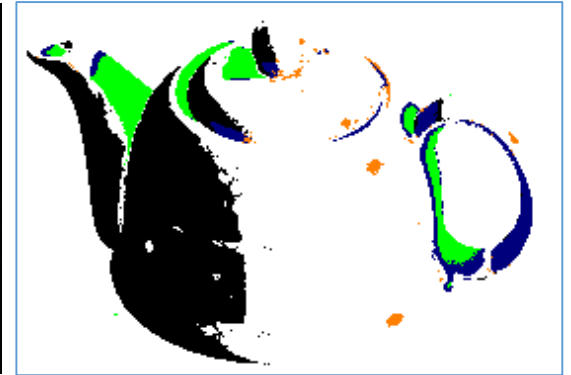
conversion  $\rightarrow$   $\mathbf{i} = \mathbf{S}^T \tilde{\mathbf{n}}$



One of the input image



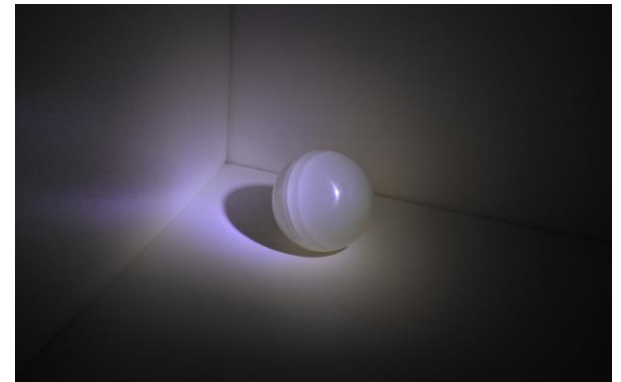
Linearized image  
(Negative: red)



Estimation of optical phenomena

# Some approaches to handle more complex scene

- Preprocess to extract pure Lambert diffuse reflection component
  - Optically or mathematically
- Use more complex model
  - Since the parameter increases, it may become unstable
- Solve in the framework of robust estimation
  - Use many input images and consider non-Lambert components as outliers
- Solve by deep learning



# Summary

- From the shading information, not only photometric information such as color and reflection properties, but also geometric information such as normal can be extracted.
- Photometric stereo cannot estimate depth. It can estimate surface normal.
- Many traditional methods assume Lambert diffuse reflection, but extended to more complex scene in recent years.