# No.5
# 反射の物理モデル
# Reflection model

担当教員：向川康博・田中賢一郎

# 3-D Scene and 2-D Image

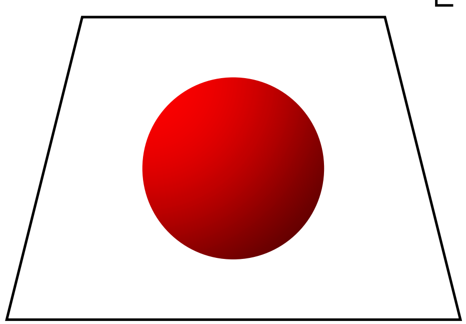
- Projection of 3-D scene to 2-D image
  - Where 2-D coordinates?
  - What colors?


3-D scene
with red ball on white desk

# Geometric Relationship

- Relation between 3-D coordinates $(x, y, z)$ of scene and 2-D coordinates $(u, v)$ of image
- Transformation by perspective projection (透視投影)

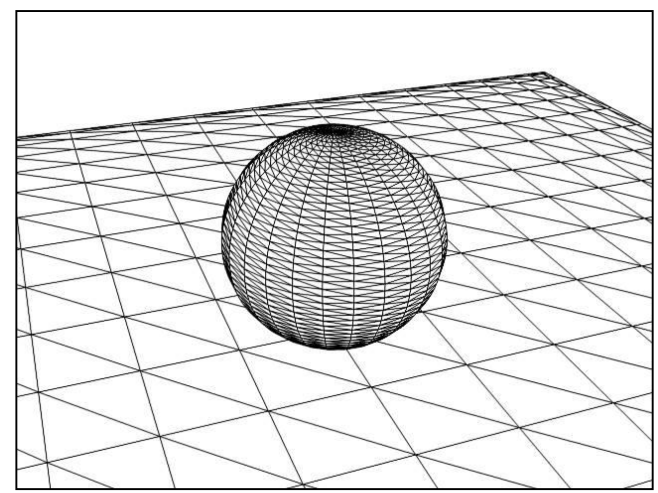
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} & P & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3-D scene

$(x,y,z)$

Geometric transformation

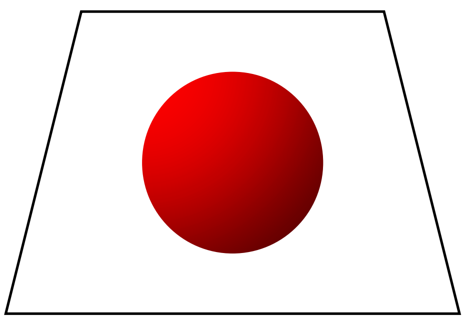


2-D image

$(u,v)$

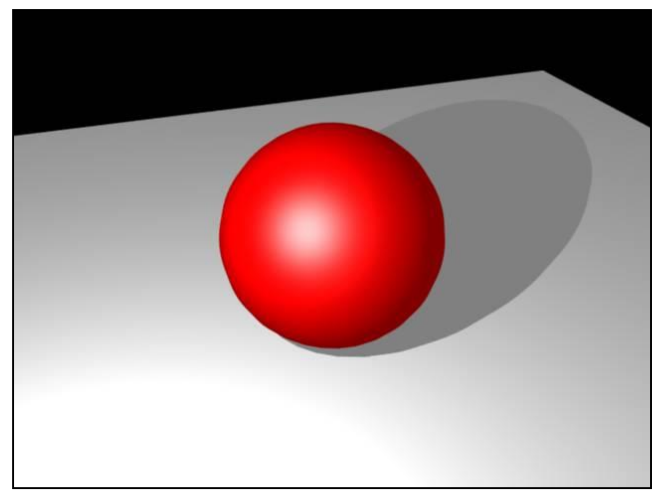
# Photometric relationship

- RGB values (intensities) of the object in the image
- Physical model for illumination and reflection
- No perfect model



3-D scene

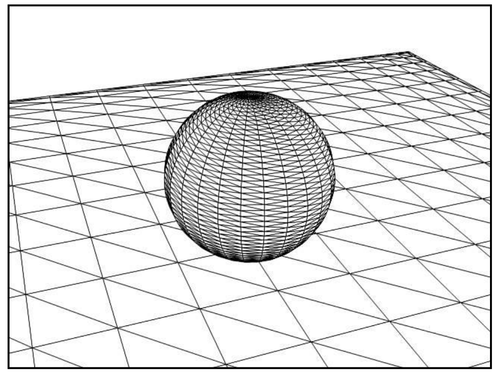
Photometric transformation

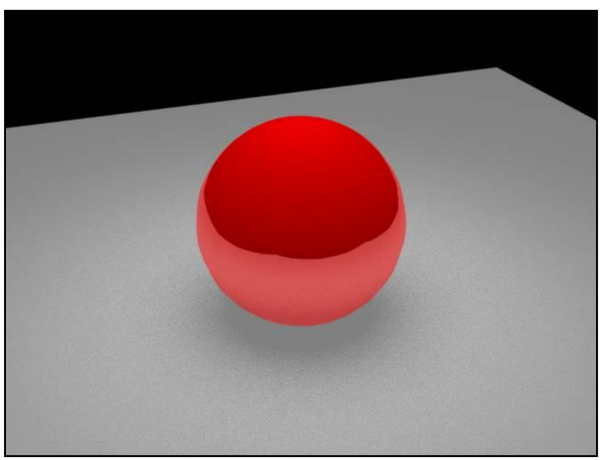


Surface color

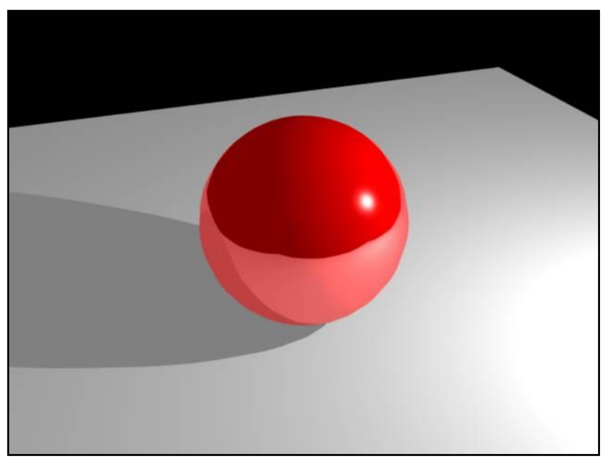
# Different Images

- Red ball on white desk

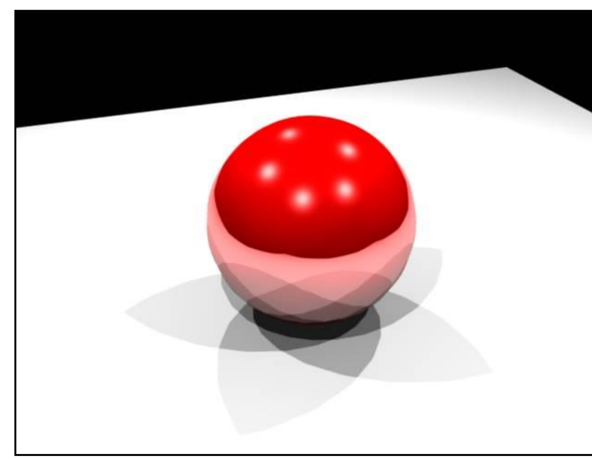




surface light source



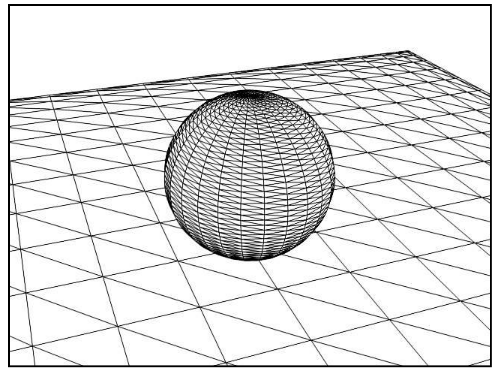
a point light source

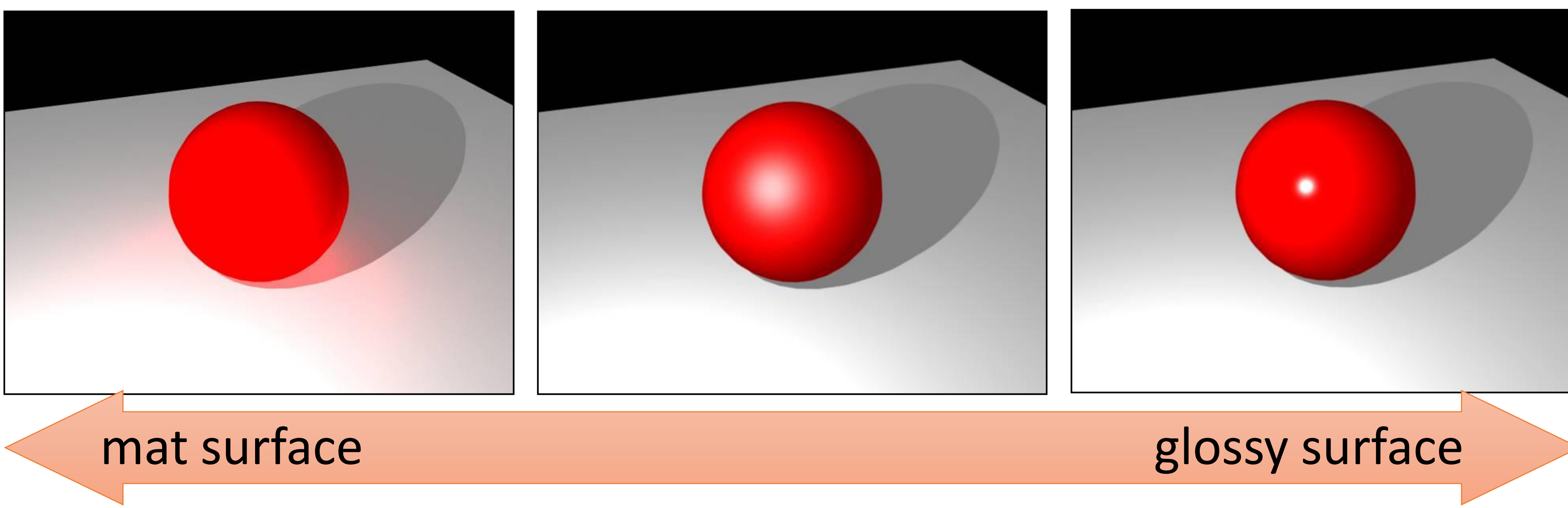


five point light sources

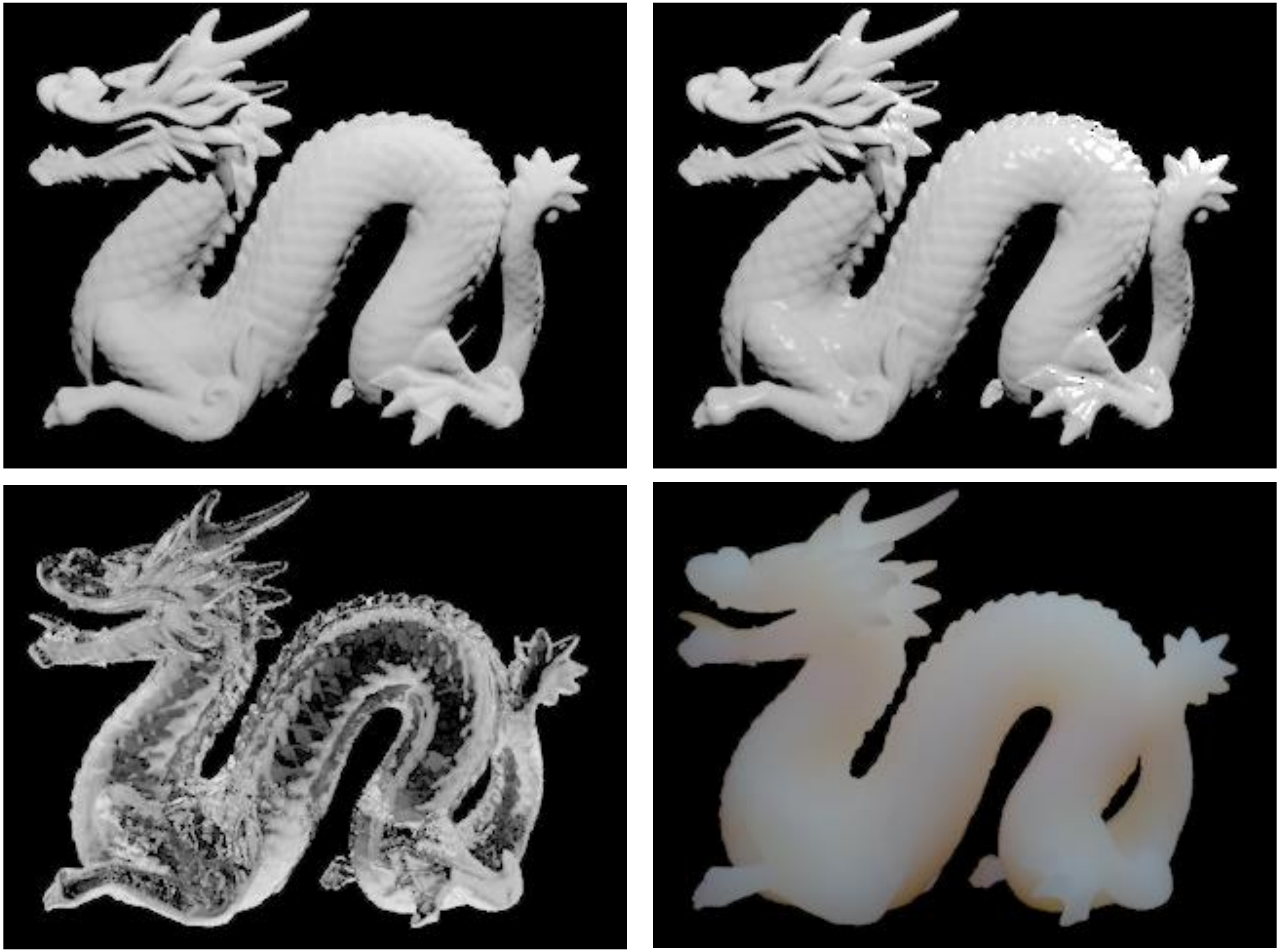
# Different images

- Red ball on white desk
- Same illumination



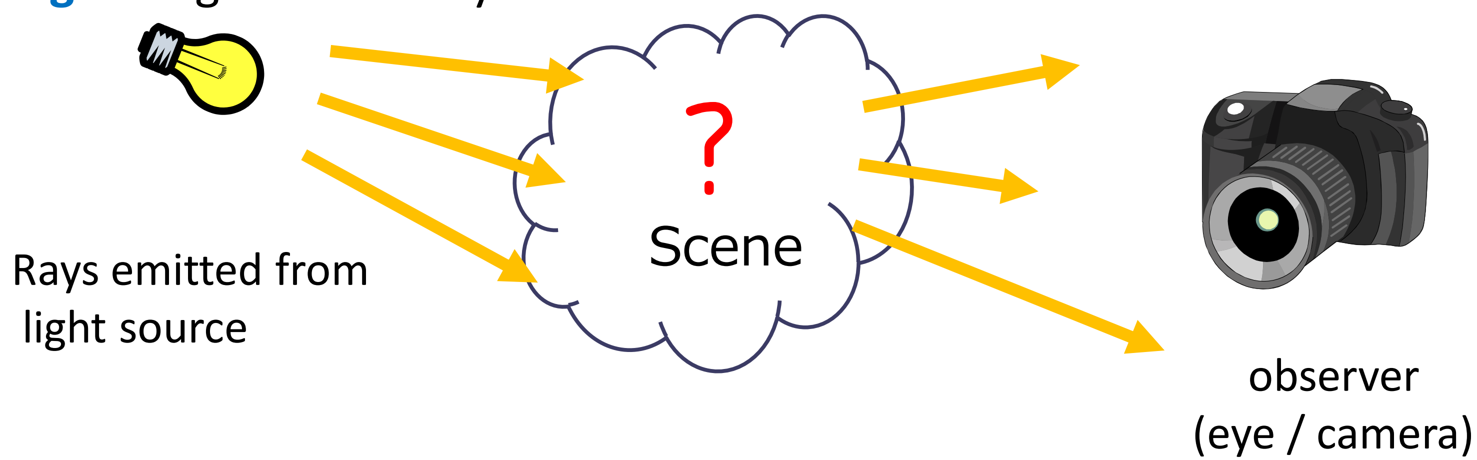


# Difference in material



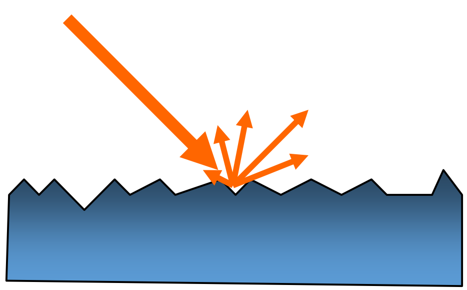
# Light transport

- Rays emitted from light source reach observer after repeating various optical phenomena such as reflection (反射), scattering (散乱), refraction (屈折), transmission (透過), interference (干渉), ...
- Light transport includes geometric and photometric properties of the scene
- Handling of **ray** rather than **image** is important
  - **Ray**: optical information before collected by lens
  - **Image**: degenerated ray in 2-D



# Accurate modeling of physical phenomenon

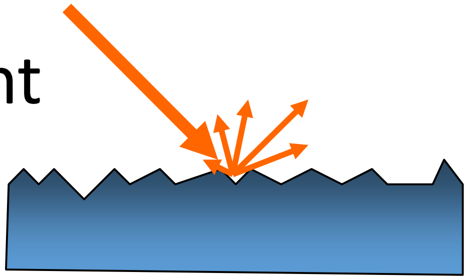
- For CG
  - Realistic rendering indistinguishable from real images
- For CV
  - Scene analysis correctly handling lighting effects
- What kind of physical phenomenon occurs when the object is illuminated?
  - geometric model: mathematics
  - photometric model: physics

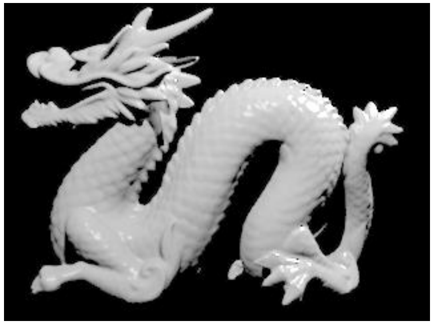


# Today's Topics

## Reflection

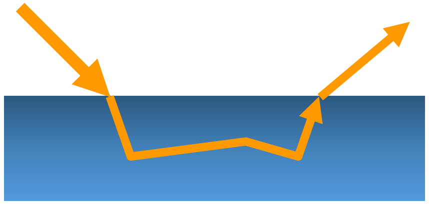
- Physical quantity of light and light transport
- Reflection model





## Scattering

- Light transport in scattering media
- Scattering model



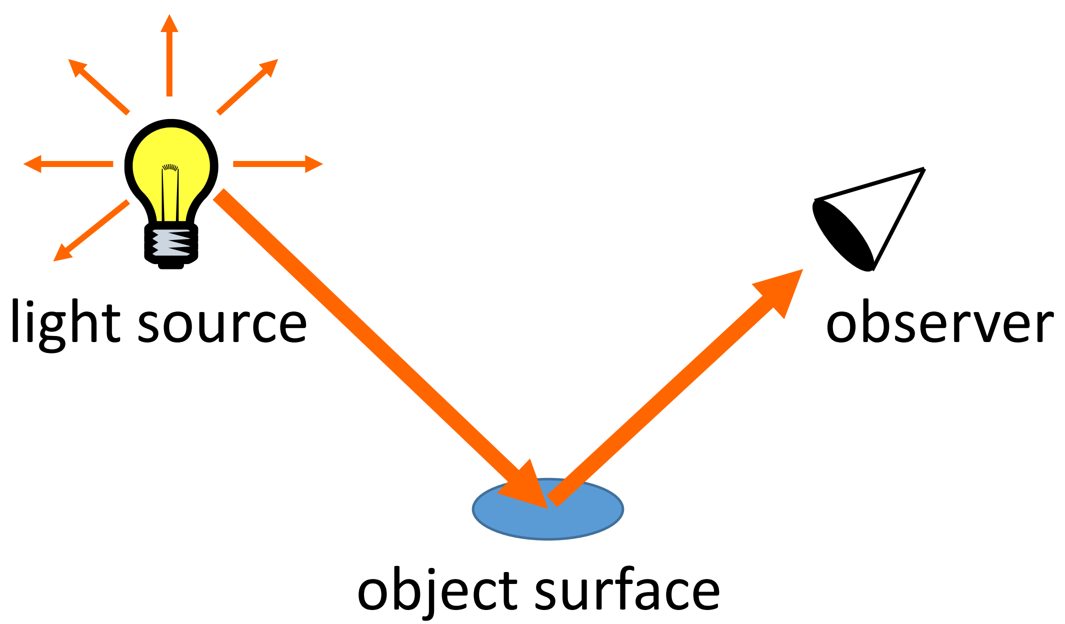
Next lecture

# Physical quantity of light
# and
# light transport

# Light energy transport

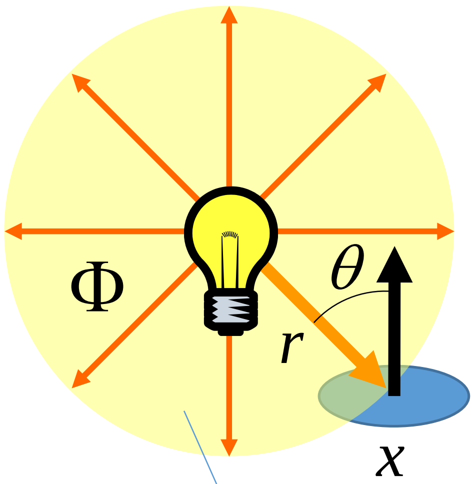
- In order to correctly treat “reflection” as a physical phenomenon,
  - Energy emitted from light source
  - Energy reaching object surface
  - Energy emitted from object surface

  should be considered.



# Light energy on object surface

- Radiant flux(放射束) : $\Phi$
  - Radiant energy per unit time
  - Unit : watt (W)
- Irradiance(放射照度) : $E(x)$
  - Light energy reaching object surface $x$
  - Radiant flux per unit area
  - Unit : $W/m^2$



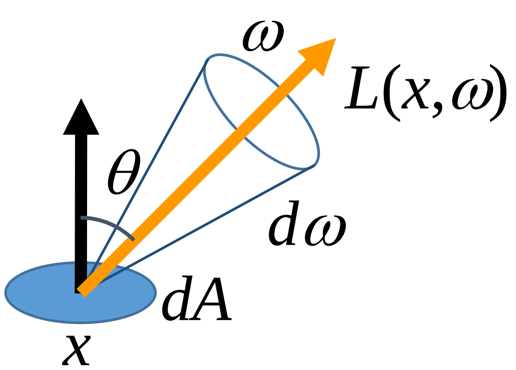

$$E(x) = \frac{\Phi \cos\theta}{4\pi r^2}$$

The received energy becomes smaller, when the light source is far and/or the surface tilts.

# Emitted light energy

- Radiance(放射輝度) : $L(x,\omega)$
  - Light energy from $x$ to $\omega$ direction
  - Radiance flux(放射束) per unit solid angle（立体角）and per unit area
  - Unit : W/m$^2$sr$^2$

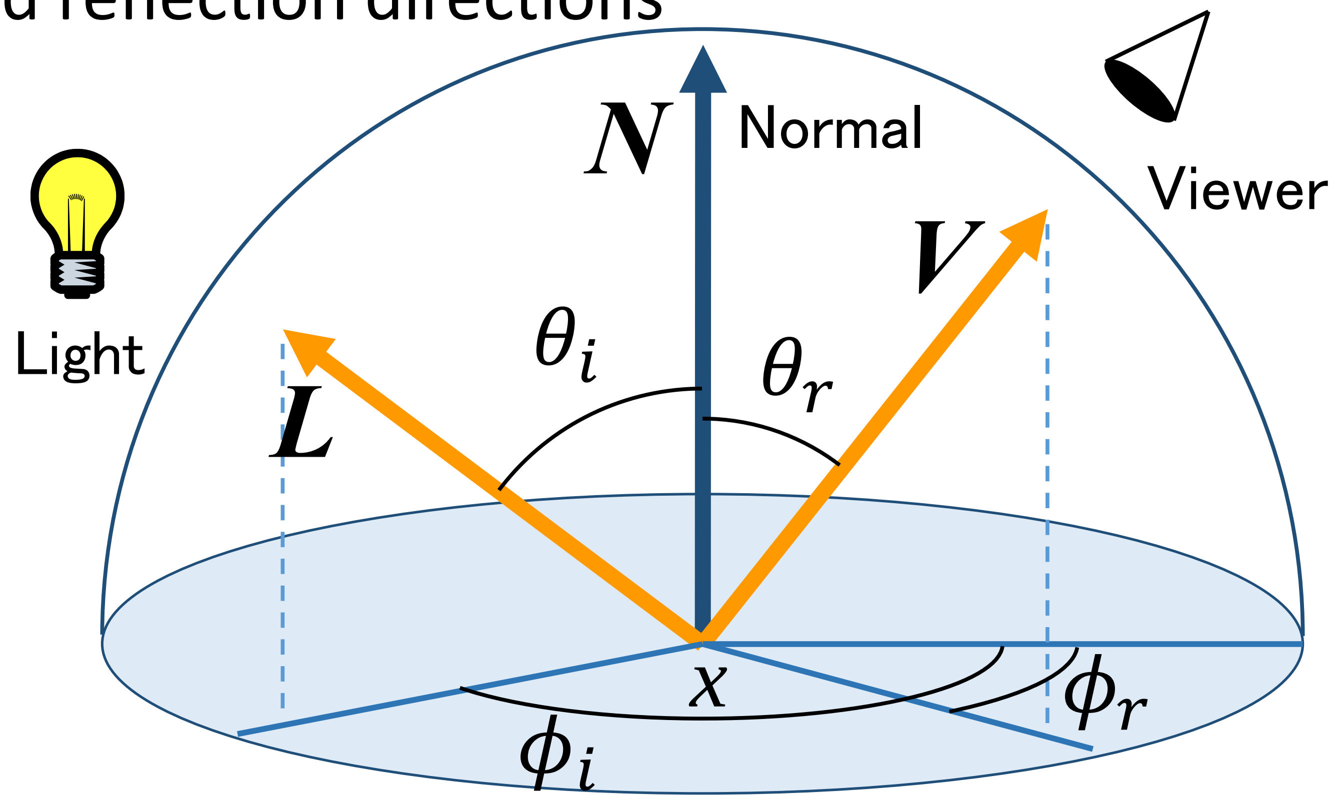
$$L(x,\omega)=\frac{d^2\Phi}{\cos\theta\, dA\, d\omega}$$




sr : steradian (unit of solid angle)

# Modeling of reflection

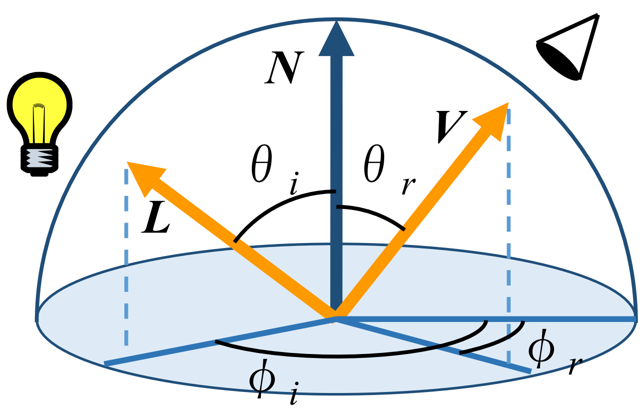
- How strongly does the light illuminated from the direction$(\theta_i, \phi_i)$ at a certain point $x$ reflects in the direction $(\theta_r, \phi_r)$ ?
- Depends on **bidirectional**（双方向）of illumination and reflection directions



# BRDF（双方向反射率分布関数）

- BRDF (**B**idirectional **R**eflection **D**istribution **F**unction)
- Ratio of radiance（出射光輝度）to irradiance（入射光照度）
- Usually, wavelength λ is omitted
  → In practice, defined by three color channels of RGB.

$$f_{BRDF}(x,\theta_i,\phi_i,\theta_r,\phi_r) = \frac{L_r(x,\theta_r,\phi_r)}{L_i(x,\theta_i,\phi_i)\cos\theta_i d\omega}$$




$$= \frac{L_r(x,\theta_r,\phi_r)}{E(x,\theta_i,\phi_i)d\omega}$$

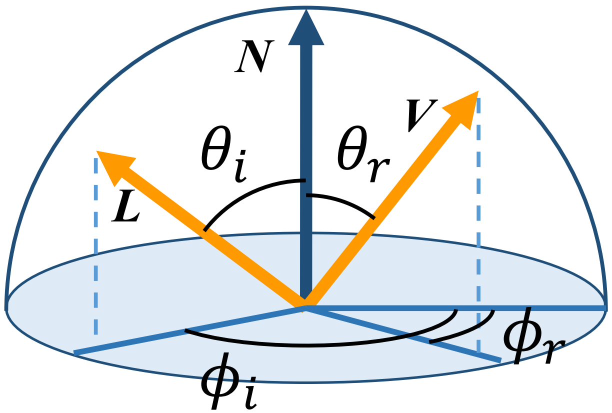
# Angle parameters of BRDF

- Anisotropic reflection（異方性反射）
  - Four angle parameters

velvet

satin

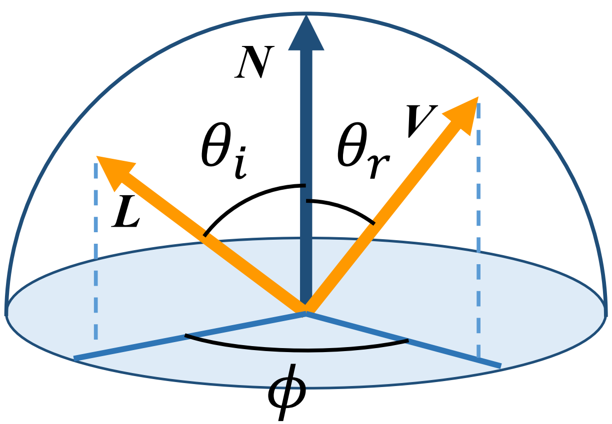
brushed metal




- Isotropic reflection（等方性反射）
  - Three angle parameters

$$f_{BRDF}(x, \theta_i, \phi_i, \theta_r, \phi_r)$$

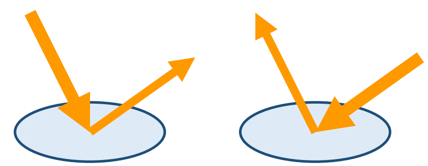
$$f_{BRDF}^{isotropic}(x, \theta_i, \theta_r, \phi)$$



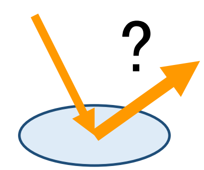
# Conditions that BRDF should satisfy

- **Condition 1：Helmholtz reciprocity(相反性)**
  - Even if illumination direction and reflection direction are exchanged, the value does not change.
  - Base for ray tracing

$$f_{BRDF}(x, L, V) = f_{BRDF}(x, V, L)$$



- **Condition 2：Law of conservation of energy (エネルギー保存の法則)**
  - Do not emit energy more than entered.




$$\int_{\Omega^+} f_{BRDF}(x, L, V)(N \cdot L)\, dL \leq 1$$

$\Omega^+$: Hemispherical surface seen from observation point

# Calculation of radiance using BRDF

- Radiance(放射輝度) of reflected light at a point $x$ on the object surface

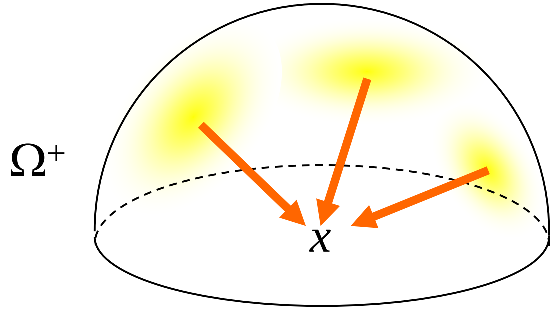
$$L_r(x,\theta_r,\phi_r) = \int_{\Omega^+} f_{BRDF}(x,\theta_i,\phi_i,\theta_r,\phi_r) L_i(x,\theta_i,\phi_i)\cos\theta_i d\omega$$

Radiance (放射輝度) — $L_r(x,\theta_r,\phi_r)$

Reflectance (反射率) — $f_{BRDF}(x,\theta_i,\phi_i,\theta_r,\phi_r)$

Irradiance (放射照度) — $L_i(x,\theta_i,\phi_i)\cos\theta_i d\omega$

$\Omega^+$: Point $x$ is illuminated from every direction on the hemisphere



# Reflection Model

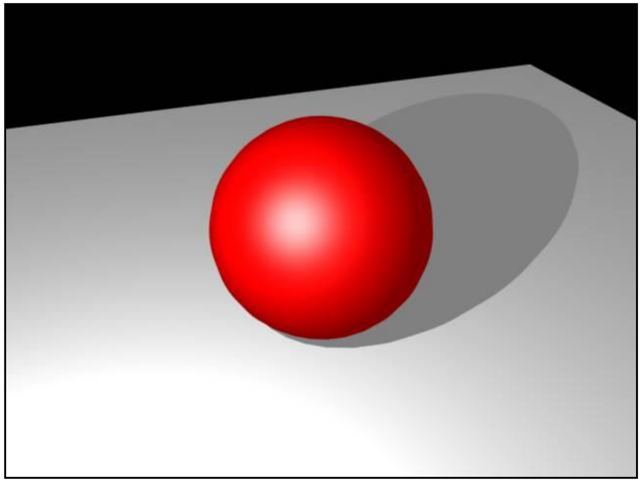
# Difference in reflection properties

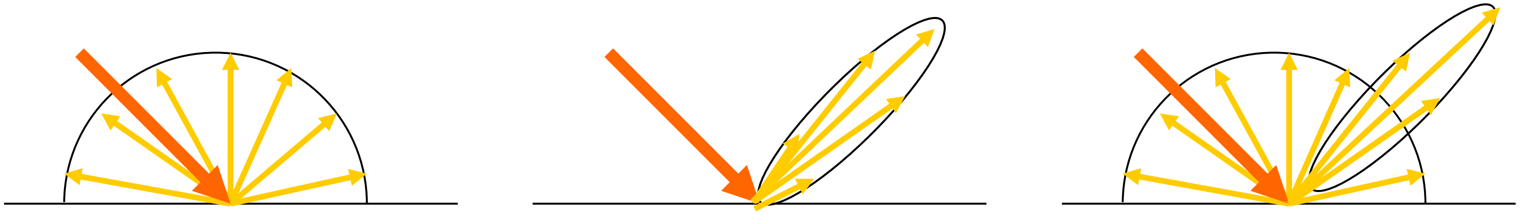
Mat ⟷ Glossy

# Dichromatic reflection model (Shafer 1985)

（2色性反射モデル）

- Reflected light = **Diffuse reflection** + **Specular reflection**
- Diffuse reflection（拡散反射）:
  - Reflection inside the surface layer
  - Object color
- Specular reflection（鏡面反射）：
  - Reflection at the border between air and surface layer
  - Light color





# Model of diffuse reflection(拡散反射)

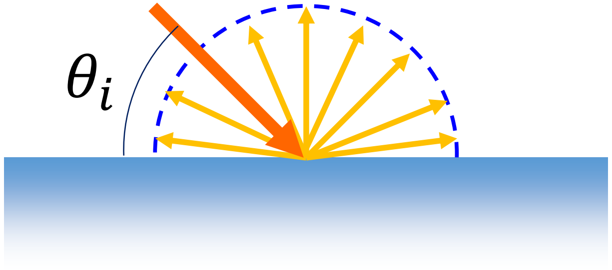
Johann Heinrich Lambert
(1728–1777)

## ■Lambert model (1760)

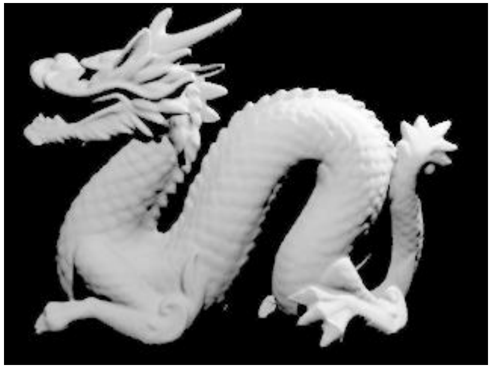
- Reflection with constant intensity in all directions
- Reflectance does not depend on illumination direction and observation direction

$$f_{BRDF}(\theta_i, \phi_i, \theta_r, \phi_r) = \rho_d$$

$$i = \rho_d \max(0, \cos\theta_i)$$

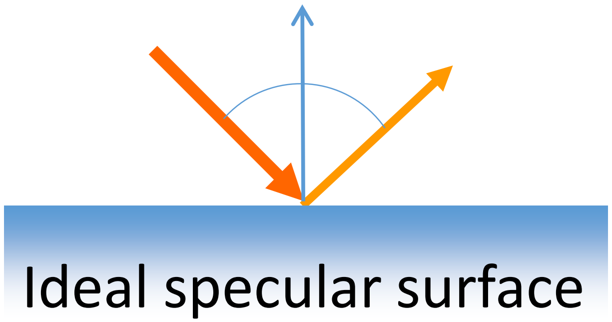


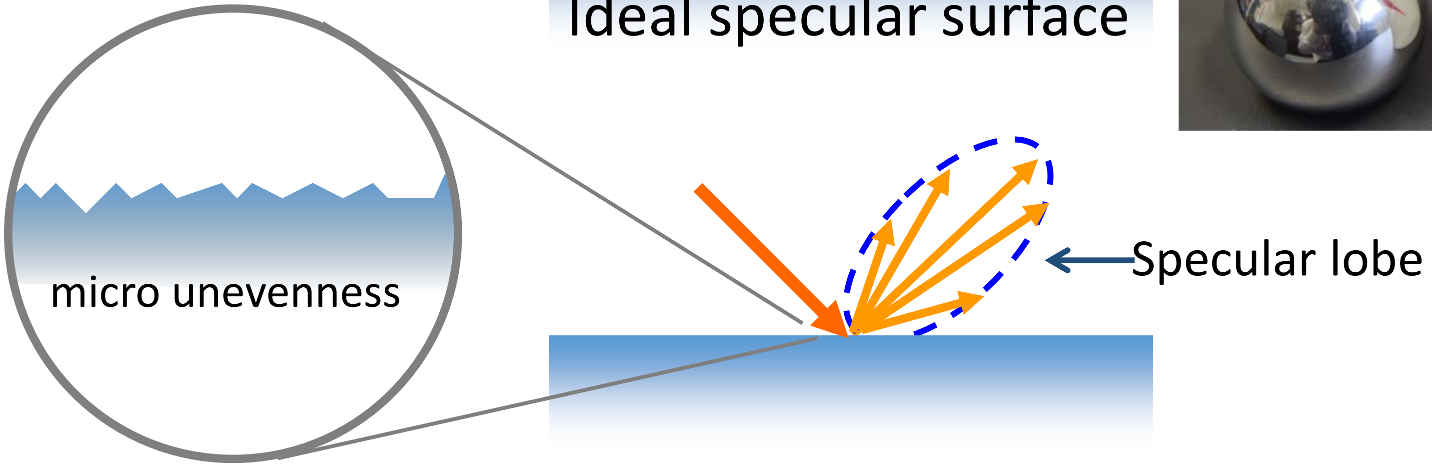

- $\rho_d$: Diffuse reflectance(拡散反射率)
- New models such as Oren-Nayar model (SIGGRAPH1994) have also been proposed, but still standard.



# Specular Reflection(鏡面反射)

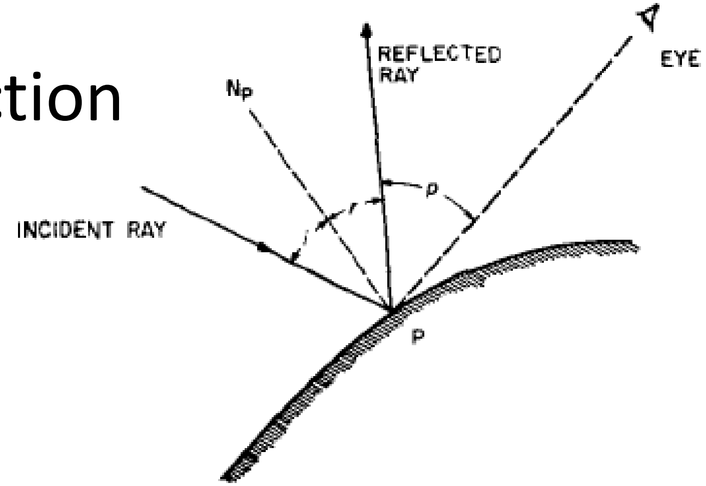
- Strongly observed in mirror direction(正反射方向)
- Due to micro unevenness on the surface, distribution becomes wider near the mirror direction.
- **Specular lobe(スペキュラーローブ)** is difficult to model accurately.

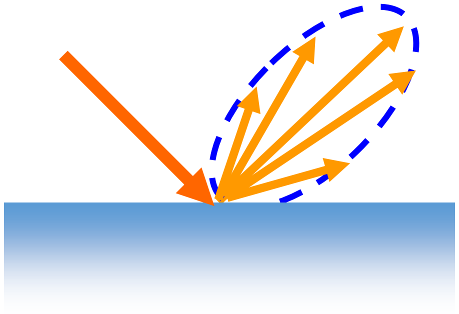


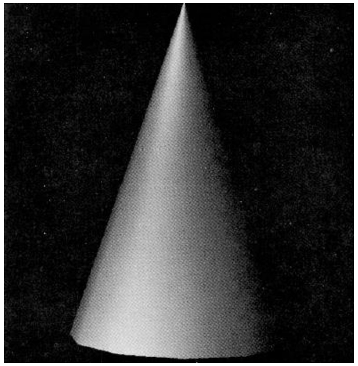



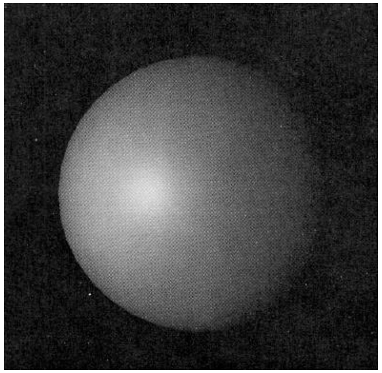
# Phong Model

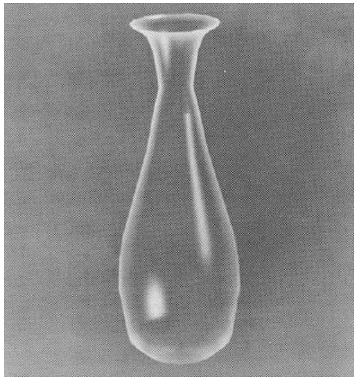
- Classical reflection model based on experience (SIGGRAPH1975)
  - It has a peak in the mirror direction
  - It weakens as angle moves away from mirror direction



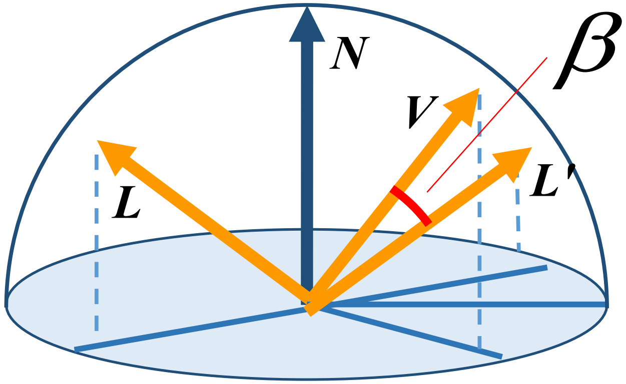






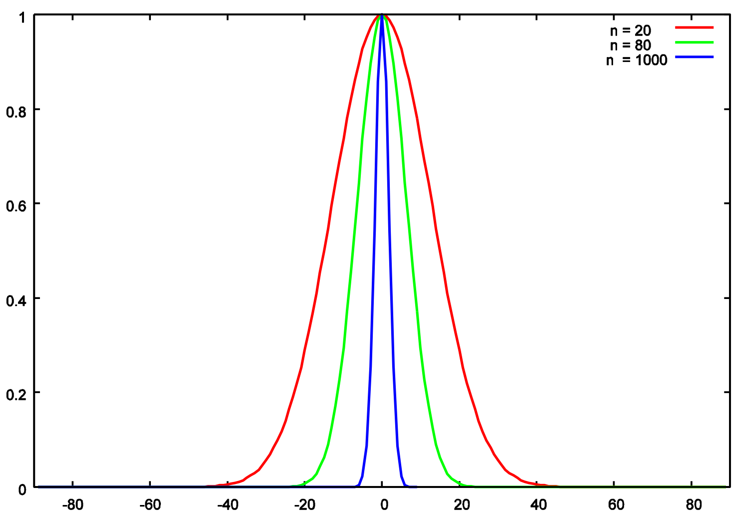
# Phong Model

- Formulation by the power cosine of the angle($\beta$) between the mirror direction($\boldsymbol{L}'$) of the light and the observation direction($\boldsymbol{V}$)



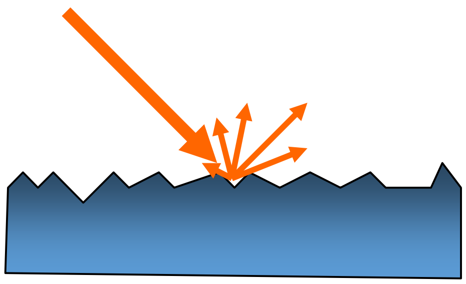

$$i = \rho_s \cos^n \beta$$

- $\rho_s$: Specular reflectance(鏡面反射率)
- $n$: Coefficient representing surface roughness
- Notice that it does not satisfy
  - Helmholtz reciprocity(相反性)
  - low of the conservation energy(エネルギー保存則)



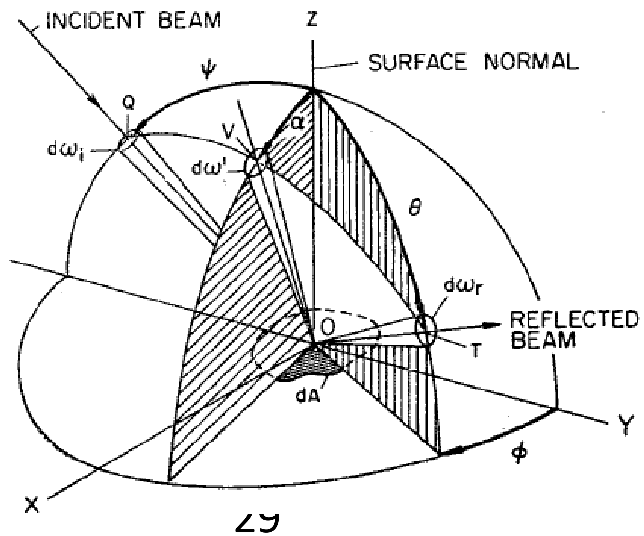
# Model based on physical analysis

- Assume that object surface is a set of micro facets（微小面）
  1. How normal vector of micro facets varies?
  2. How surface point is occluded due to surface roughness?
  3. How Fresnel reflection(フレネル反射) effects?



# Torrance-Sparrow Model

- A model based on the physical analysis which was developed earliest in the optical field (JOSA1967)
  - Modeling occlusion by micro facets and Fresnel reflection
- Represent off-specular(オフスペキュラー)
  - The peak of the specular reflection moves from the mirror direction
  - Title is "Theory for Off-Specular Reflection From Roughened Surfaces"



# Formulation by Blinn (SIGGRAPH1977)

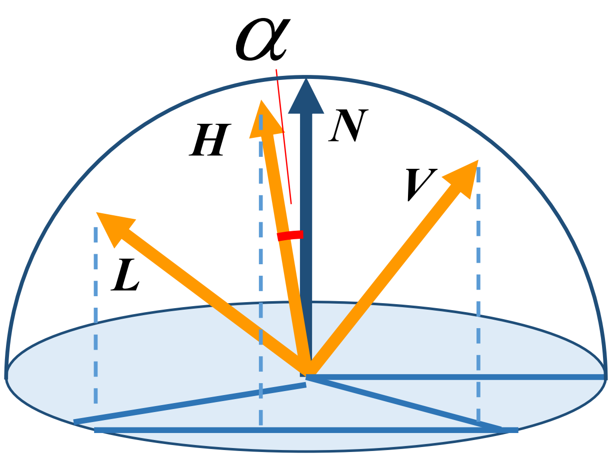
- Redefine Torrance-Sparrow model and apply to CG

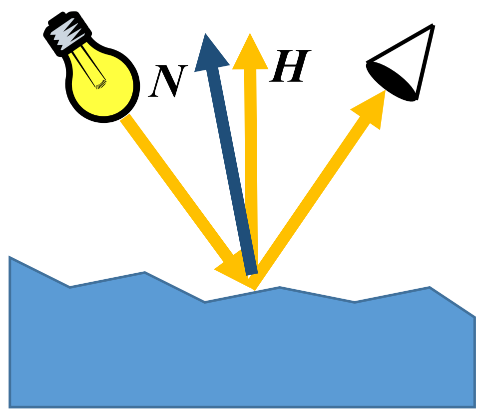
$$i = \rho_s \frac{DGF}{\mathbf{N} \cdot \mathbf{V}}$$

- $D$: Distribution function(法線分布)
  - Representing the variation of the surface normal
- $G$: Geometrical attenuation factor (幾何減衰)
  - Representing self-occulusion
- $F$: Fresnel reflection (フレネル反射)
  - Representing Fresnel reflections at boundary of different refractive indexes (屈折率)

# $D$: Distribution Function(法線分布)

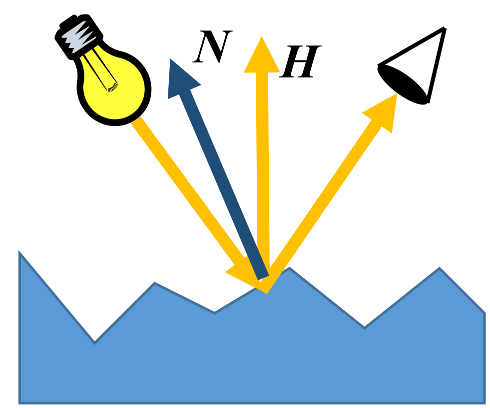
- A probability density function(確率密度関数) of an angle $\alpha$ formed by a half vector ($\boldsymbol{H}$) and a normal direction($\boldsymbol{N}$)
  - Half vector: bisector direction of the illumination and the observation directions
  - Assuming a set of micro facets that produce perfect specular reflection
  - How much do the normal vary to the half vector?




**Smooth surface**

$\boldsymbol{N}$ and $\boldsymbol{H}$ tend to coincide




**Rough surface**

$\boldsymbol{N}$ and $\boldsymbol{H}$ tend not to coincide

# Various Distribution Functions (法線分布)

- Redefinition of the Phong model using half vector

$$D_1 = \cos^{n_1} \alpha$$

- Gauss distribution used in Torrance-Sparrow model

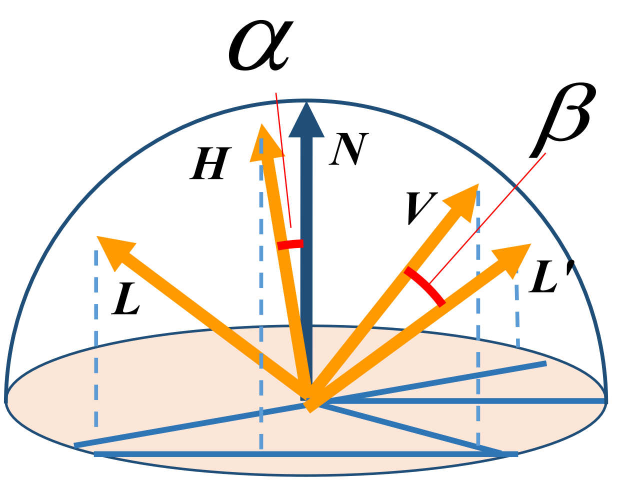
$$D_2 = e^{-(\alpha n_2)^2}$$

- Trowbridge-Reitz model

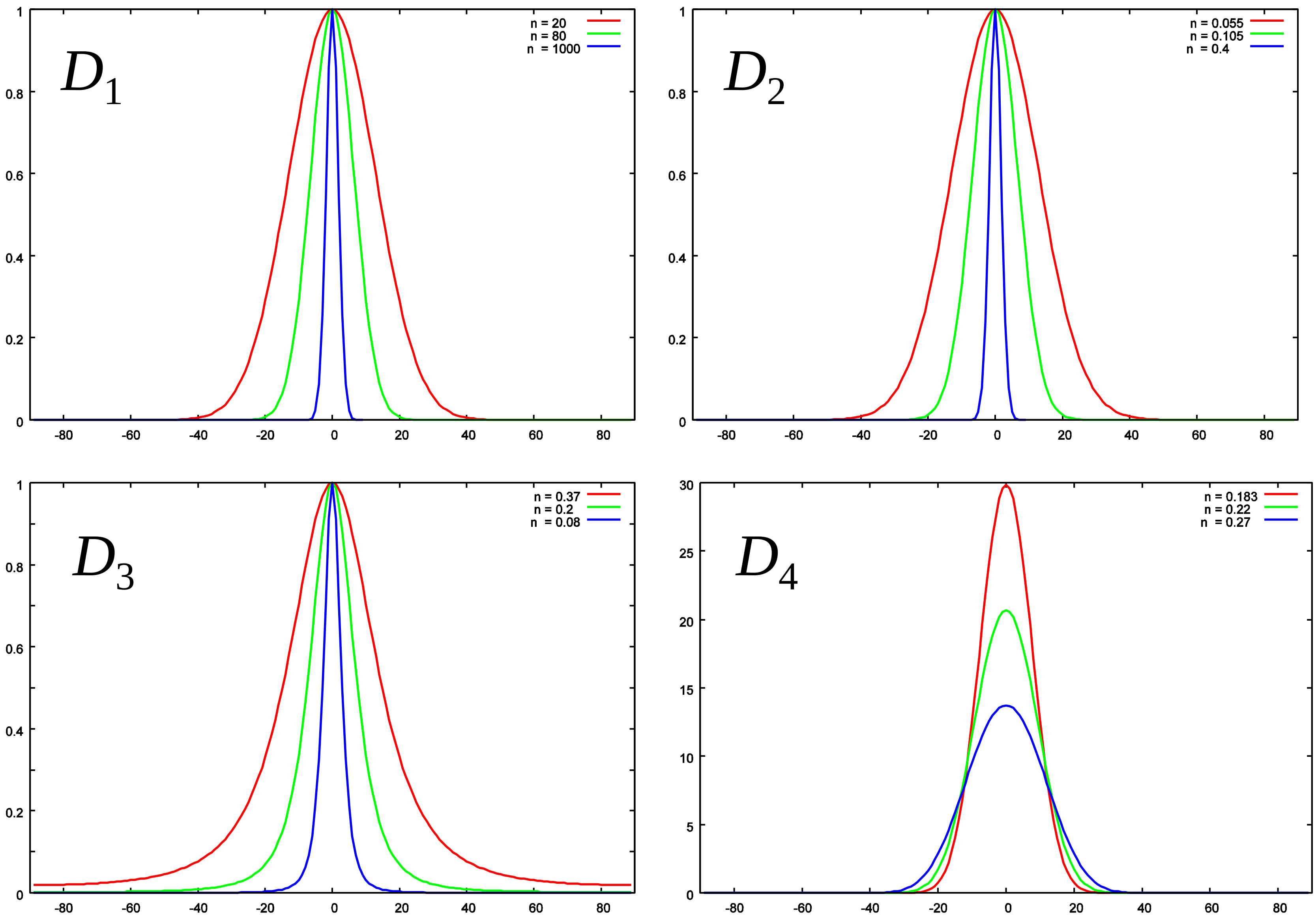
$$D_3 = \left( \frac{(n_3)^2}{\cos^2 \alpha((n_3)^2 - 1) + 1} \right)^2$$

- Cook-Torrance model（Beckman distribution）

$$D_4 = \frac{1}{(n_4)^2 \cos^4 \alpha} e^{-\left( \frac{\tan^2 \alpha}{(n_4)^2} \right)}$$



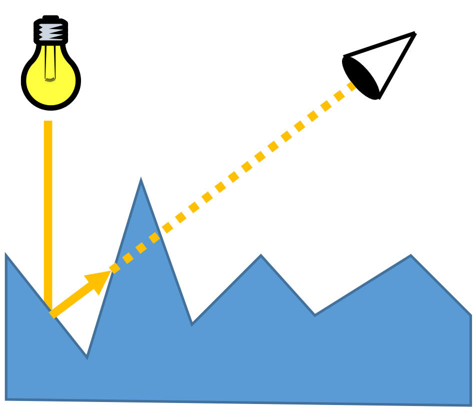
# Examples of distribution function



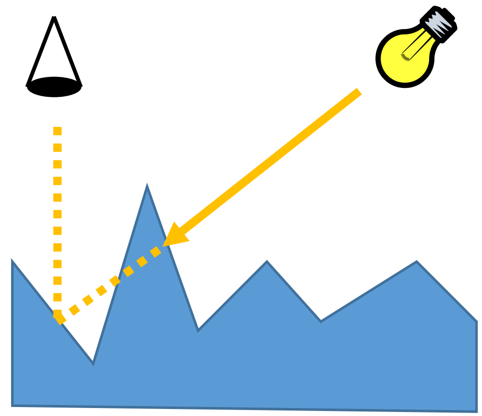
# $G$: Geometrical Attenuation Factor (幾何減衰)

- Self-masking(自己遮蔽) and self-shadowing(自己陰影) caused by irregularities of micro facets
- As the illumination direction and/or observation direction approach tangent plane, attenuation increases

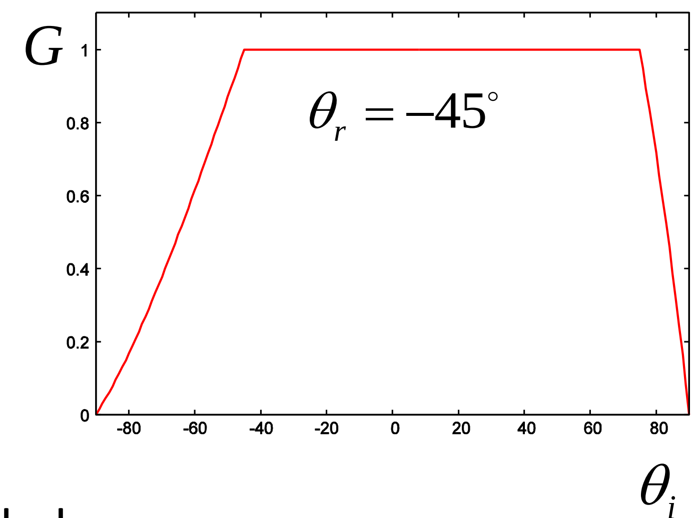
$$G = \min\left(1, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{V})}{\mathbf{V}\cdot\mathbf{H}}, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{L})}{\mathbf{V}\cdot\mathbf{H}}\right)$$



**Masking:**
Reflected light is occluded

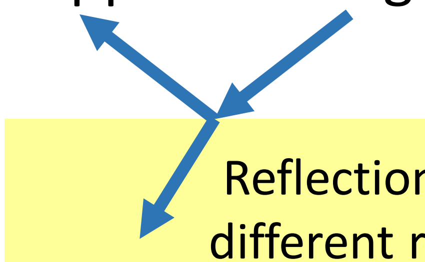


**Shadowing:**
Incident light is occluded



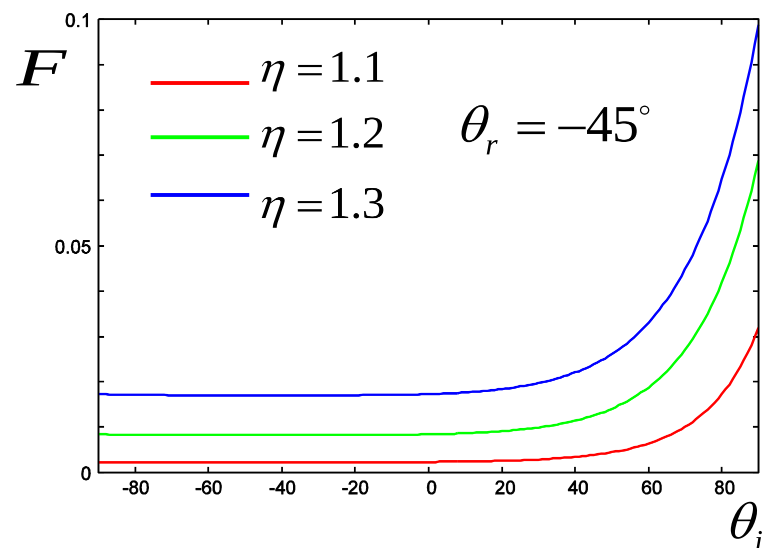
# $F$: Fresnel Reflection(フレネル反射)

- Represent Fresnel reflection
  - Reflectance changes with refractive index(屈折率) and angle
  - As the illumination direction and/or observation direction approach tangent plane, reflectance becomes higher




Approximate expression:

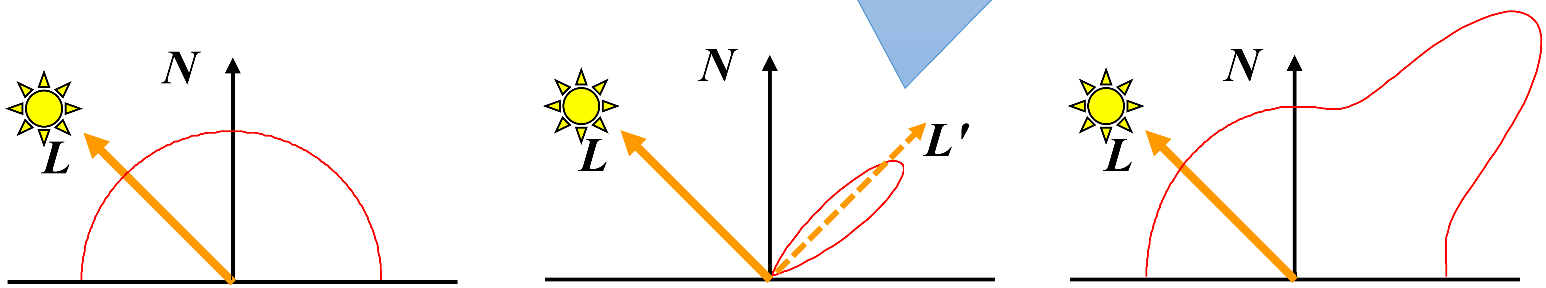
$$F = \frac{1}{2}\left\{ \frac{\sin^2(\theta_i - \theta_r)}{\sin^2(\theta_i + \theta_r)} + \frac{\tan^2(\theta_i - \theta_r)}{\tan^2(\theta_i + \theta_r)} \right\}$$

# Example of Torrance-Sparrow Model

- When illumination direction $\theta_i$=45

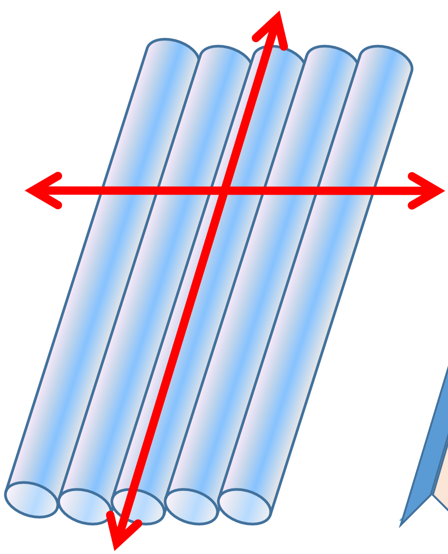
Off-specular
Peak at $\theta_r$=47



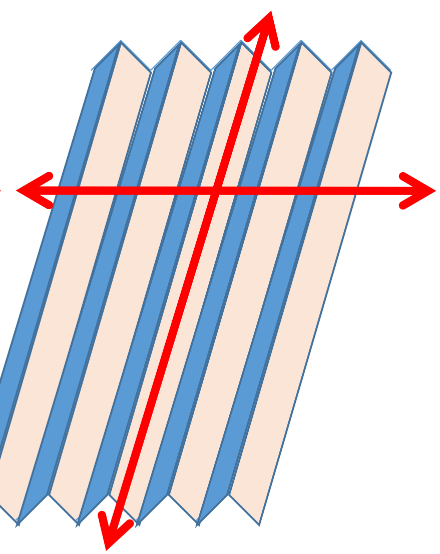

Diffuse reflection    Specular reflection    Sum of both reflection

# Ward Model (SIGGRAPH1992)

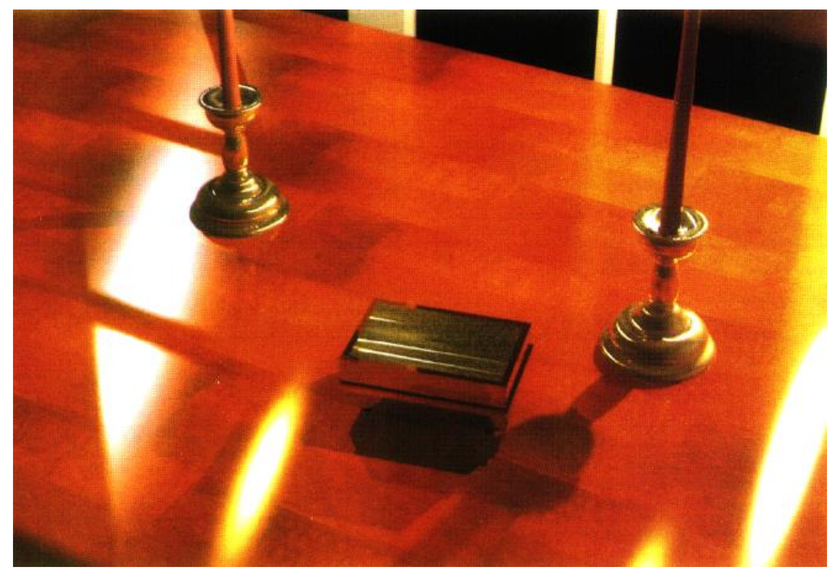
- Representing anisotropic reflection(異方性反射)
  - Extension of distribution function in the Torrance-Sparrow model
  - Different roughness coefficients for parallel and vertical directions to the axis (fiber or brushing direction)



cloth



brushed metal

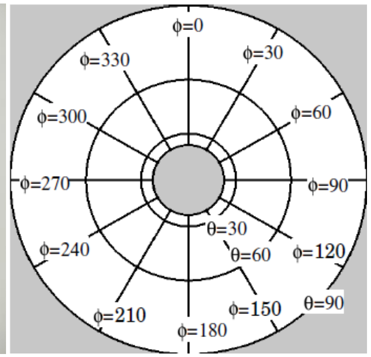


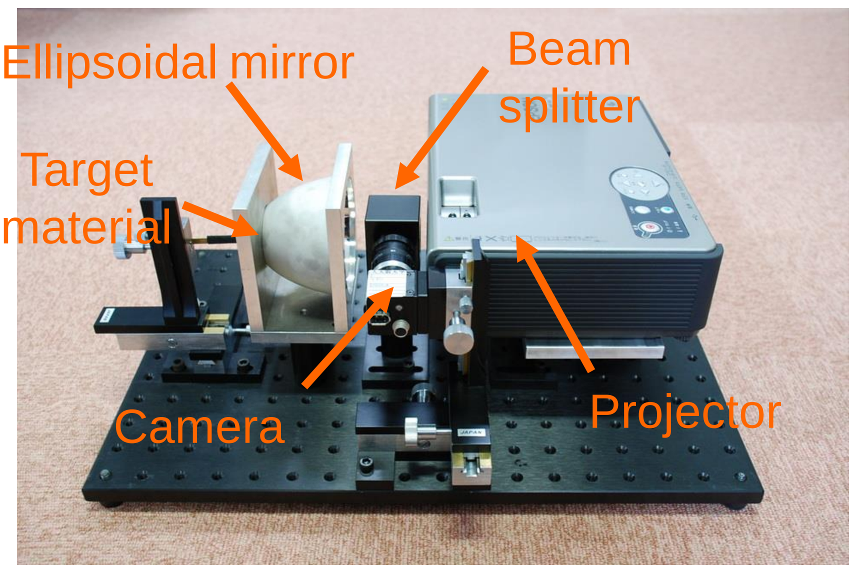
# MERL BRDF Database

- Matusik et al., A Data-Driven Reflectance Model, ACM Transactions on Graphics (2003)
- Densely measured BRDFs of 100 different materials
  - plastic, metal, fabric, rubber, marble, ...
- Spherical target

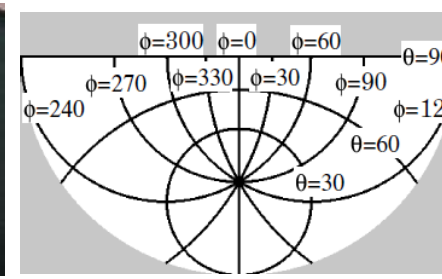
# BRDF sampling devices

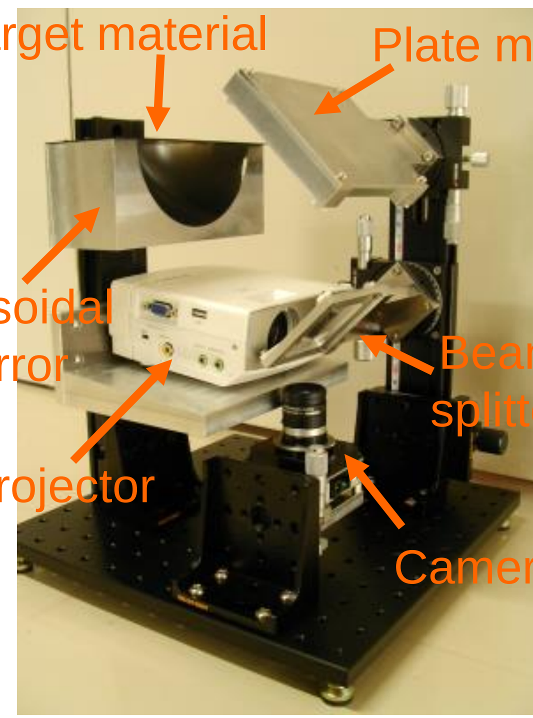
- Vertical setup (RCG-1)

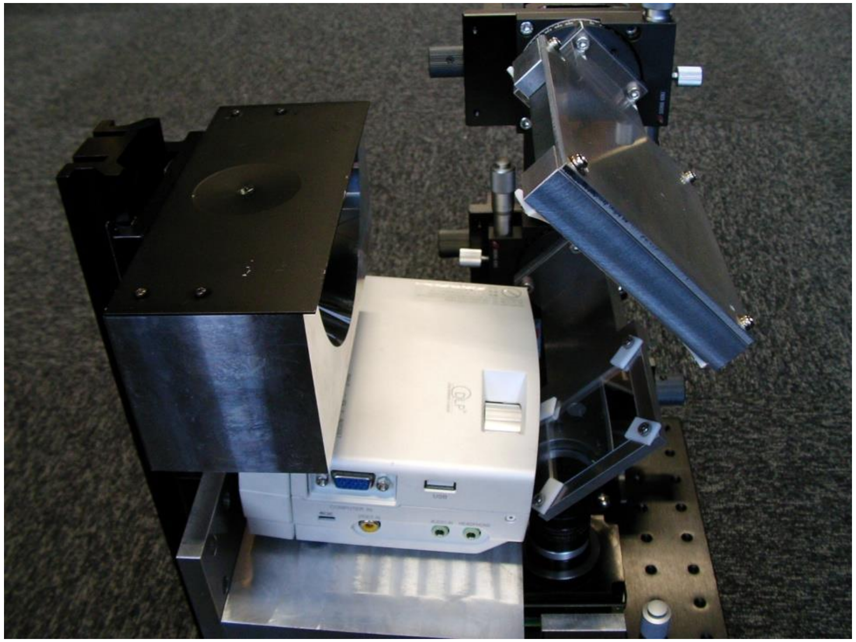


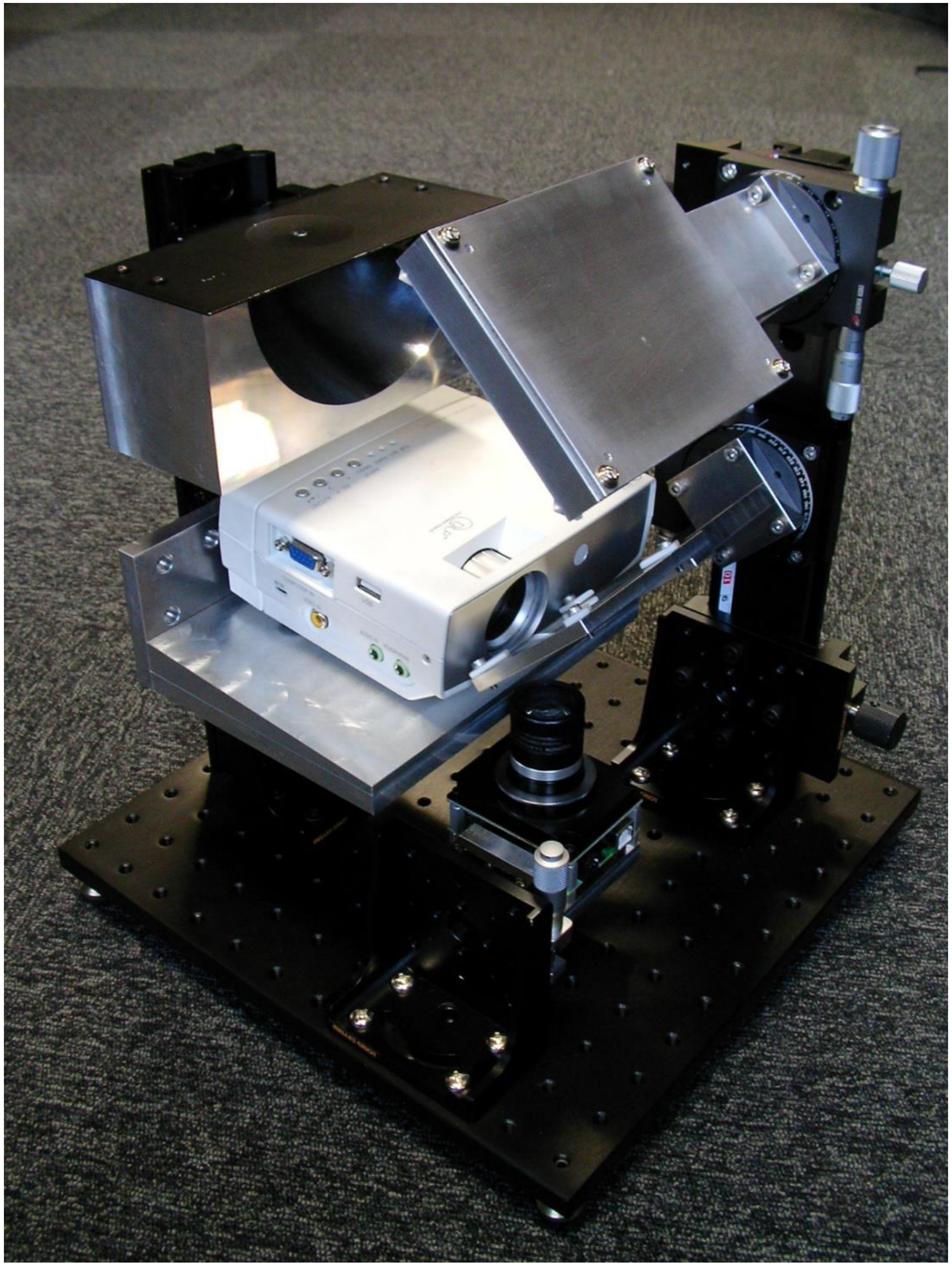




- Horizontal setup (RCG-2)



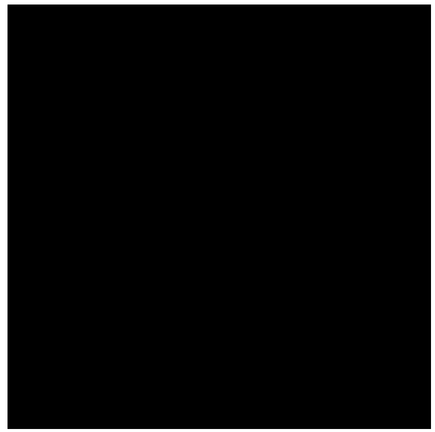







# Sampled BRDF for CG


Real coin



Sampled BRDF

Geometric shape

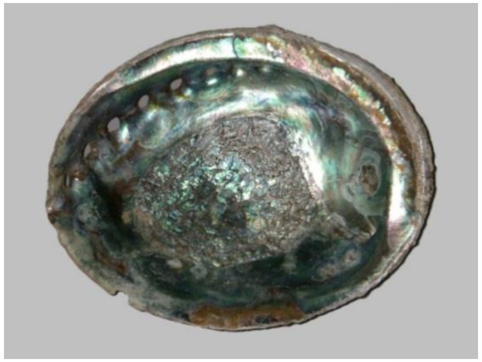
- isotropic reflection
- per one degree

# Structural color（構造色）

■Complex physical model

Multilayer interference
（多層膜干渉）

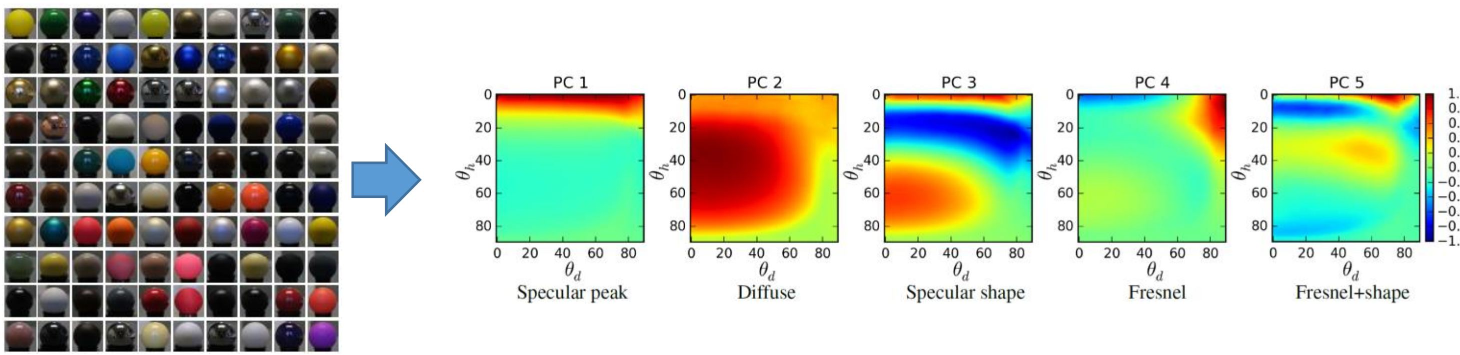
Structural color of nature



Mexican shell

# Sparse sampling + PCA

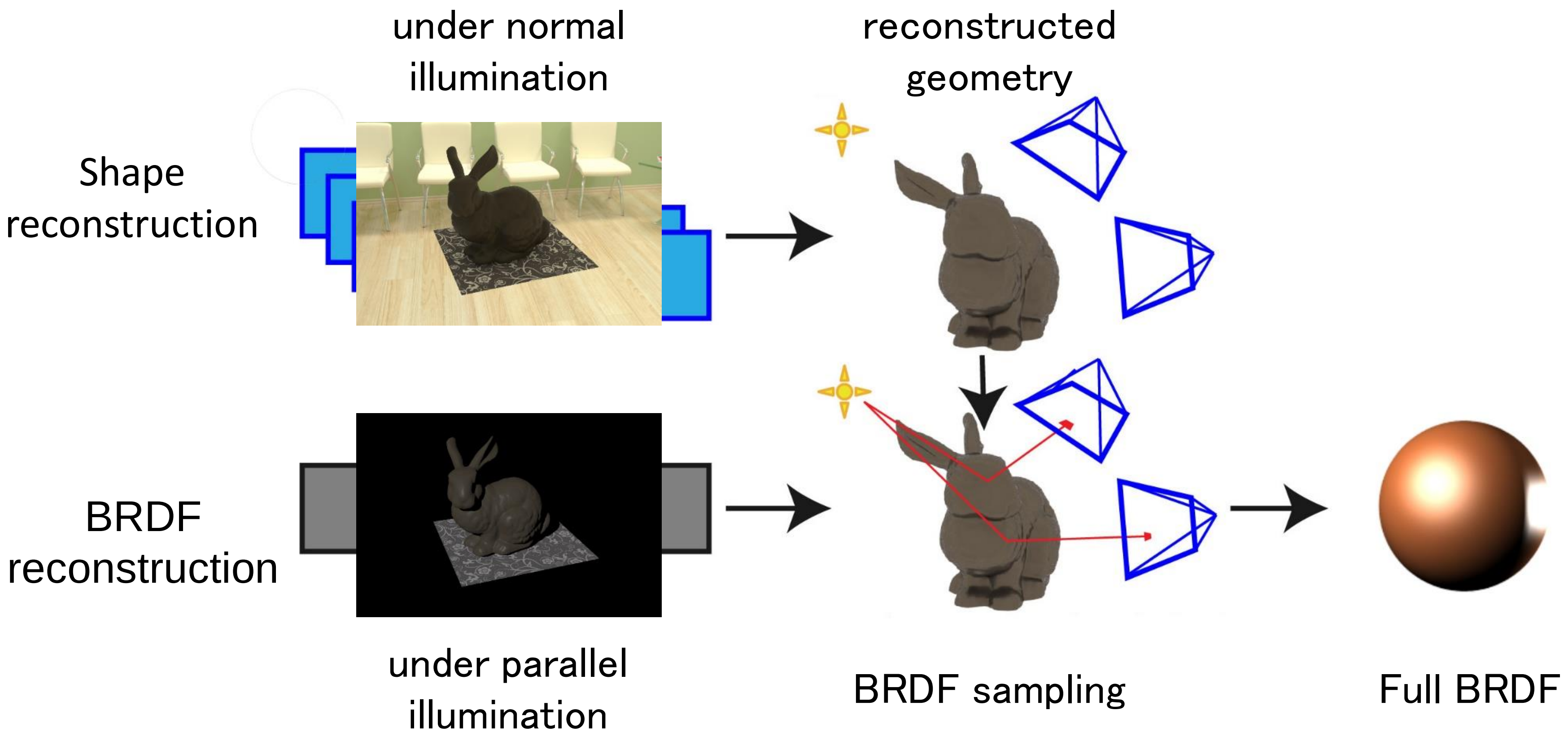
- PCA of MERL BRDF database
  - The BRDF of most objects can be represented by a linear sum of a small number of bases (BRDF is sparse)
  - BRSD measurement is equivalent to estimation of coefficients.




  - any BRDF= $\Sigma c_i$ base BRDF(i)

Nielsen et al., On Optimal, Minimal BRDF Sampling for Reflectance Acquisition, SIGGRAPH Asia 2015

# BRDF sampling from real object




T. Ono, H. Kubo, T. Funatomi, Y. Mukaigawa, ``BRDF Reconstruction from Real Object using Reconstructed Geometry of Multi-view Images'', Proc. SIGGRAPH Asia2017.

T. Ono, H. Kubo, T. Funatomi, Y. Mukaigawa, ``BRDF Reconstruction from Real Object using Reconstructed Geometry of Multi-view Images'', Proc. SIGGRAPH Asia2017.

# The Result of Simulated Experiment

# Summary

- The early papers are still active.
  - diffuse reflection: 1760
  - specular reflection: 1967
- Recently, complete measurement of BRDF becomes possible.

Light Stage: University of Southern California

# Final report

- Explain advantages and disadvantages to use complex and realistic BRDF model for CG and CV

| | advantage | disadvantage |
|---|---|---|
| CG | ■ Physical phenomena are faithfully reproduced.<br>■ Realistic image which is distinguishable from real photo can be rendered. | ■ High computational cost.<br>■ Difficult parameter settings. |
| | **advantage** | **disadvantage** |
| CV | ■ Complex phenomenon can be treated.<br>■ Image analysis in uncontrolled environment. | ■ Unstable model fitting.<br>■ Sometimes ill-posed problem. |