No.5
反射の物理モデル
Reflection model

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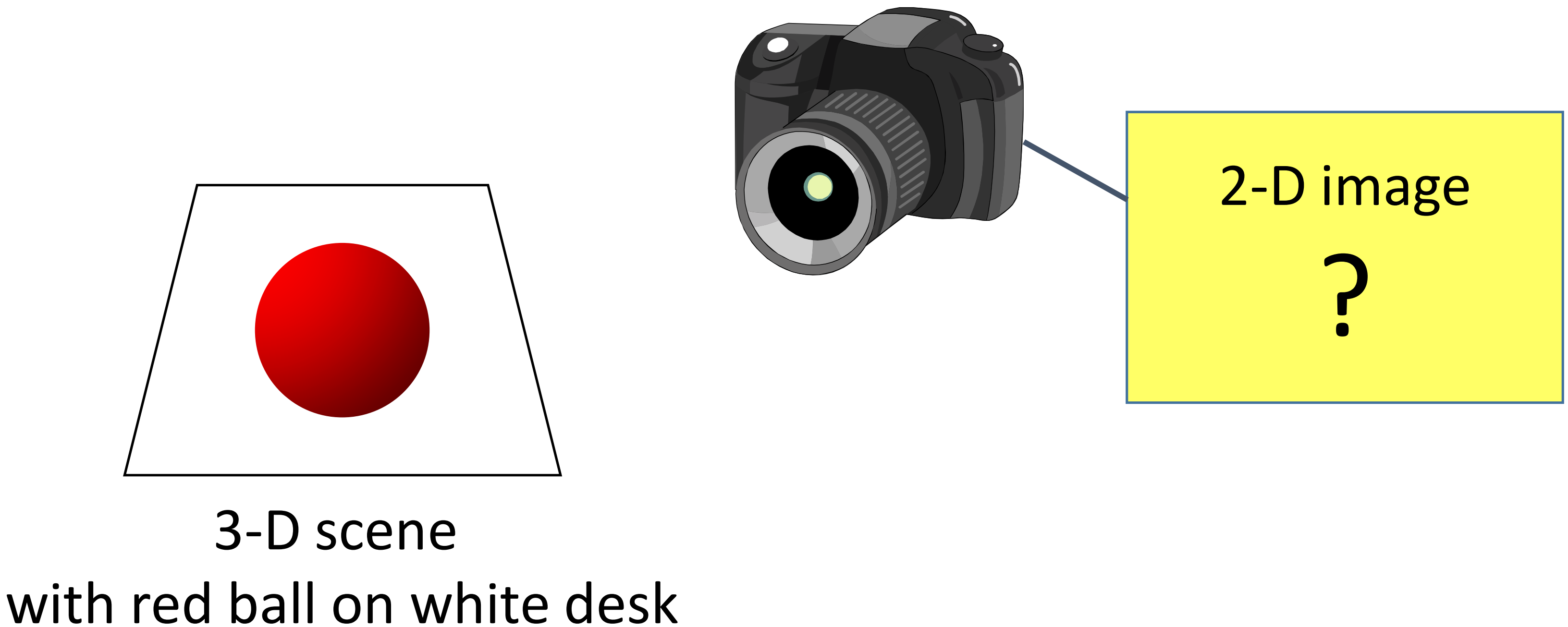
3-D Scene and 2-D Image

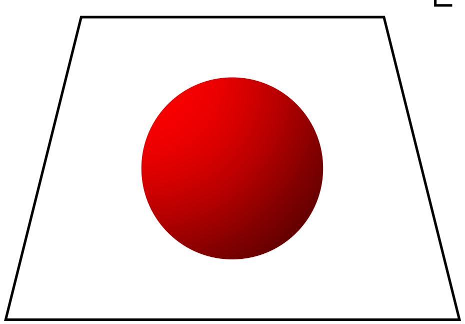
- Projection of 3-D scene to 2-D image
  - Where 2-D coordinates?
  - What colors?



Geometric Relationship

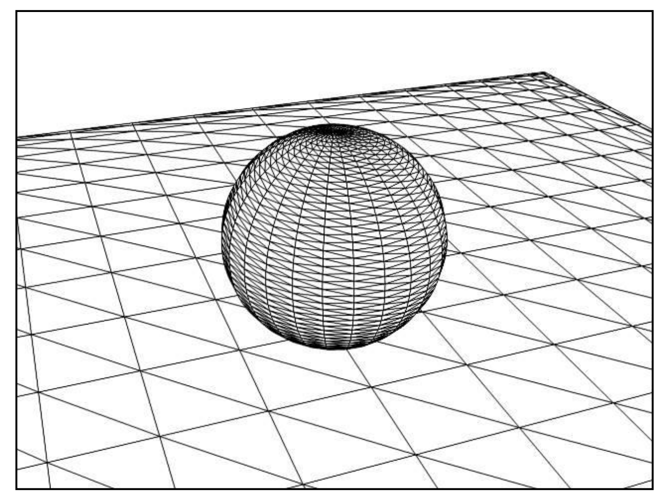
- Relation between 3-D coordinates (x, y, z) of scene and 2-D coordinates (u, v) of image
- Transformation by perspective projection (透視投影)

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} & P & \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3-D scene
(x,y,z)

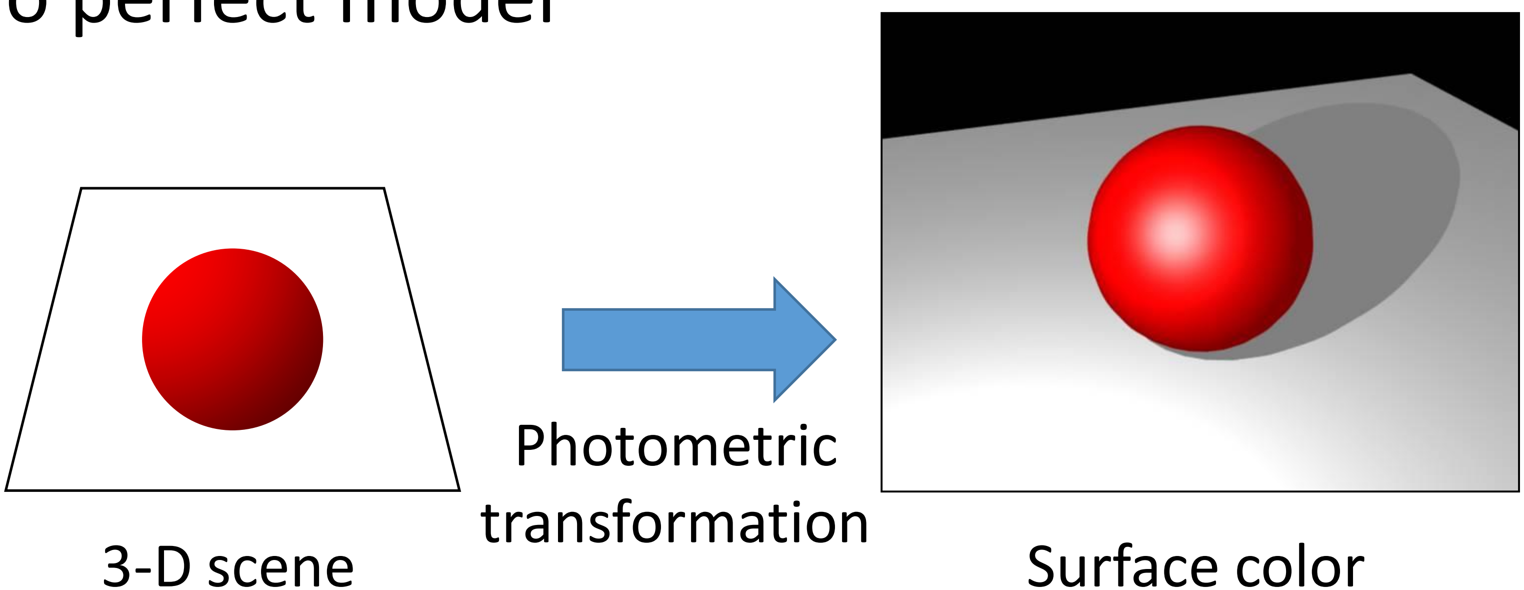
Geometric transformation



2-D image
(u,v)

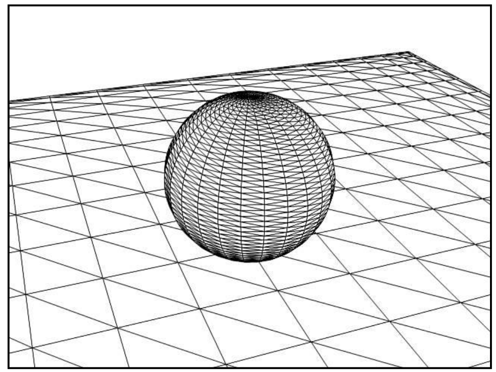
Photometric relationship

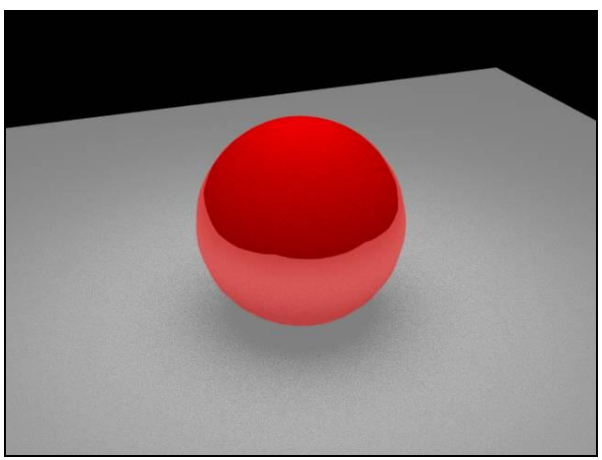
- RGB values (intensities) of the object in the image
- Physical model for illumination and reflection
- No perfect model

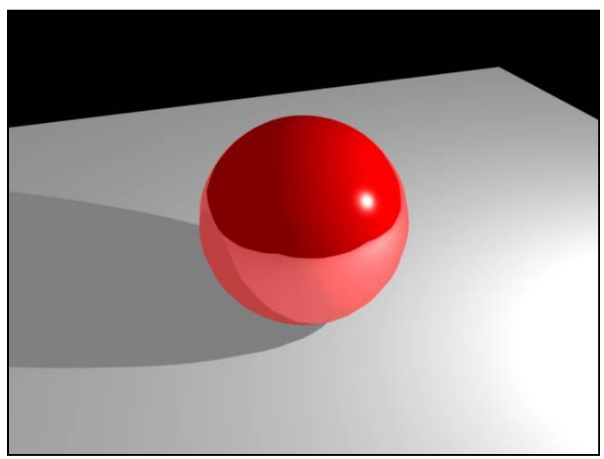


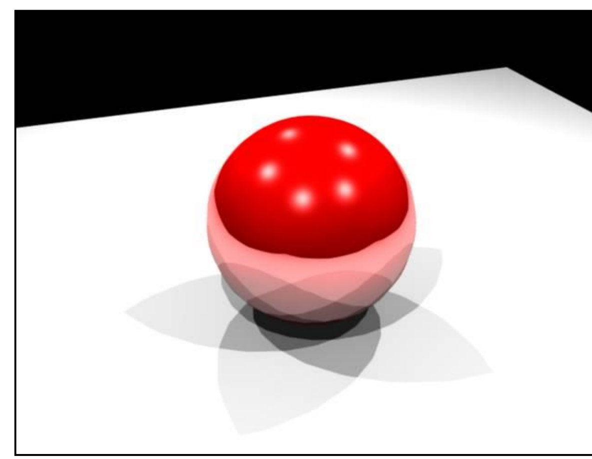
Different Images

- Red ball on white desk





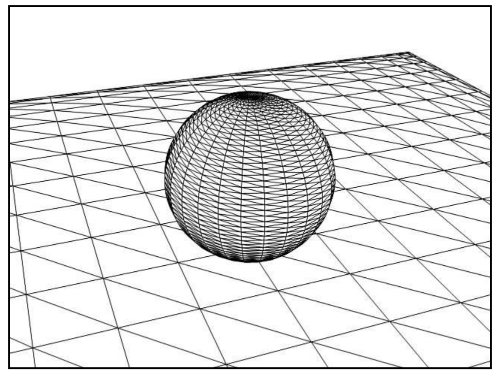


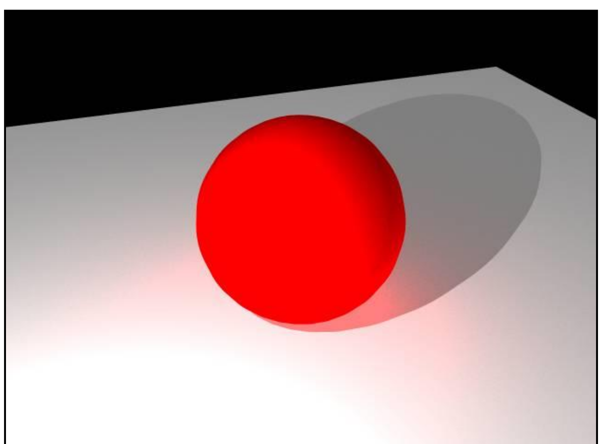


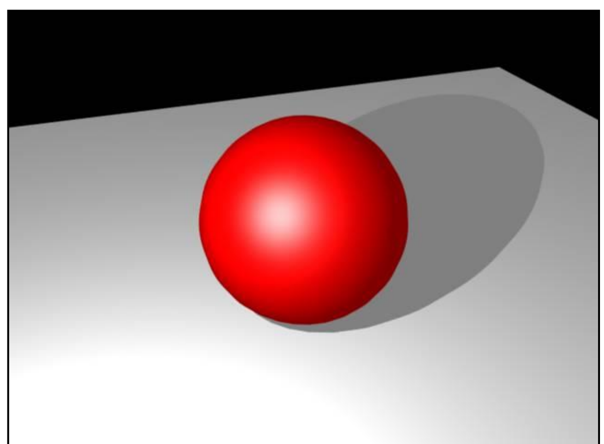
mini-report1: What is the difference?

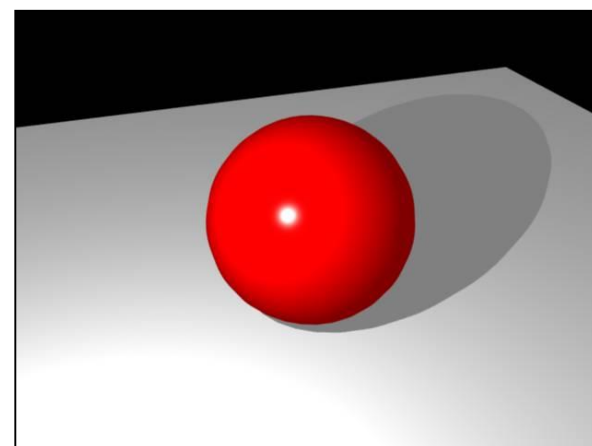
Different images

- Red ball on white desk
- Same illumination



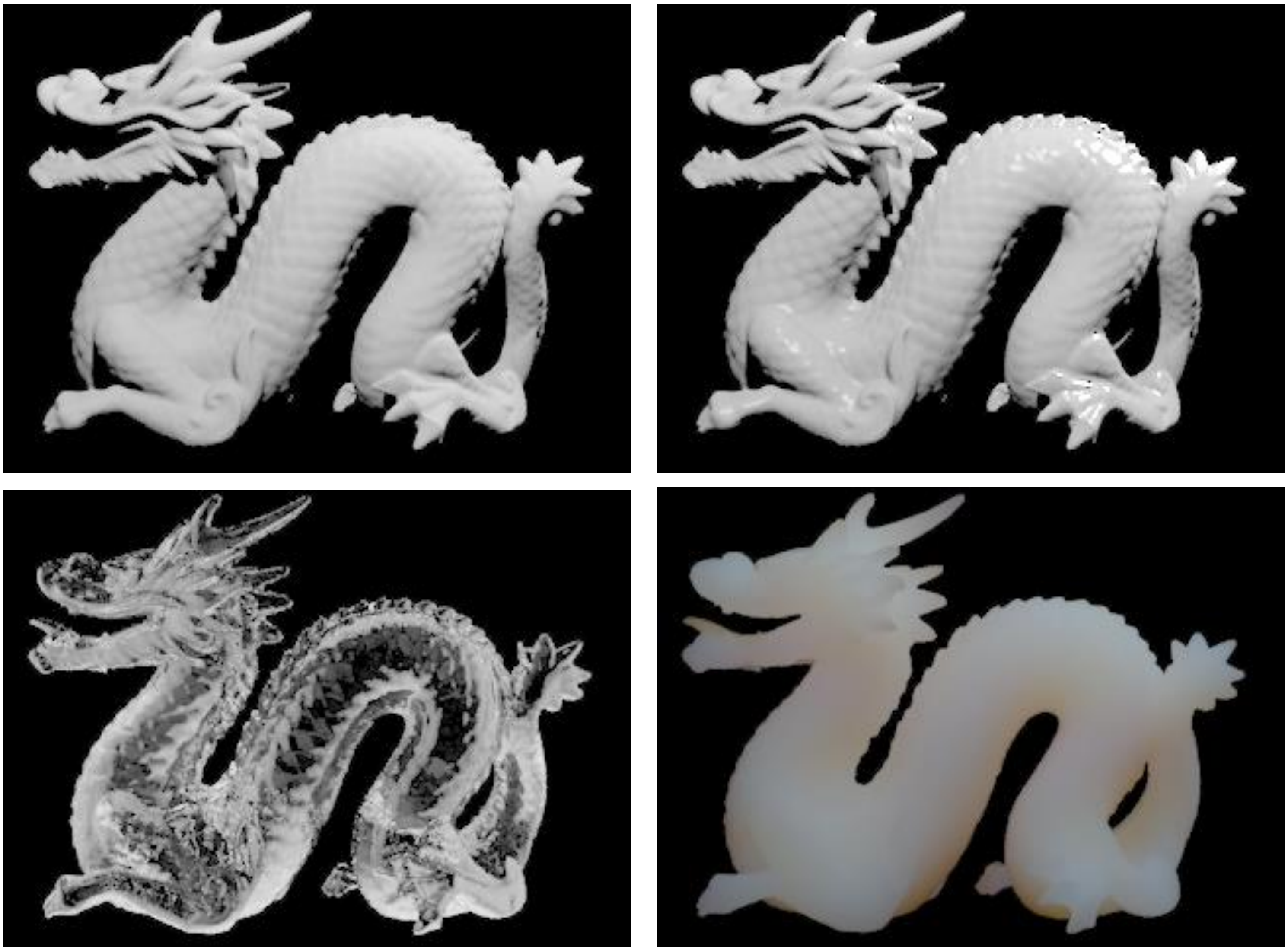






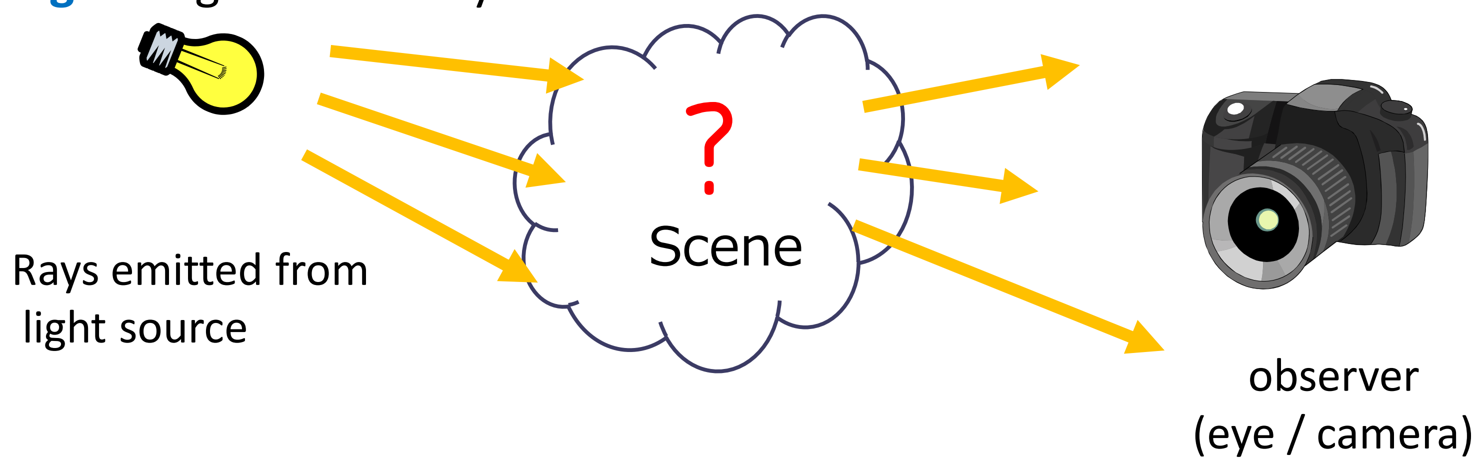
mini-report2: What is the difference?

Difference in material



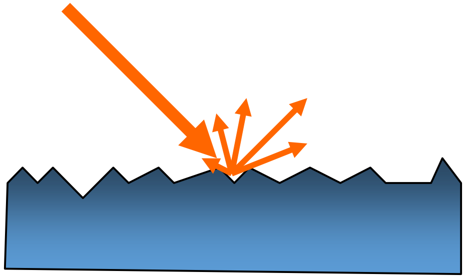
Light transport

- Rays emitted from light source reach observer after repeating various optical phenomena such as reflection (反射), scattering (散乱), refraction (屈折), transmission (透過), interference (干渉), ...
- Light transport includes geometric and photometric properties of the scene
- Handling of **ray** rather than **image** is important
  - **Ray**: optical information before collected by lens
  - **Image**: degenerated ray in 2-D



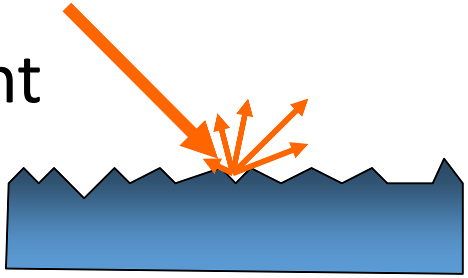
Accurate modeling of physical phenomenon

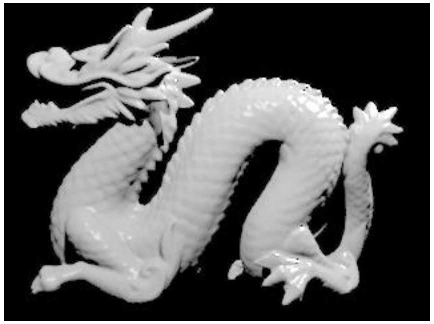
- For CG
  - Realistic rendering indistinguishable from real images
- For CV
  - Scene analysis correctly handling lighting effects
- What kind of physical phenomenon occurs when the object is illuminated?
  - geometric model: mathematics
  - photometric model: physics



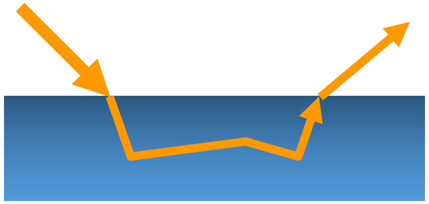
Today's Topics

- **Reflection**
  - Physical quantity of light and light transport
  - Reflection model





- **Scattering**
  - Light transport in scattering media
  - Scattering model



Next lecture

Final report

- Explain advantages and disadvantages to use complex and realistic BRDF model for CG and CV

<table>
<tr><td rowspan="2">CG</td><td>advantage</td><td>disadvantage</td></tr>
<tr><td></td><td></td></tr>
<tr><td rowspan="2">CV</td><td>advantage</td><td>disadvantage</td></tr>
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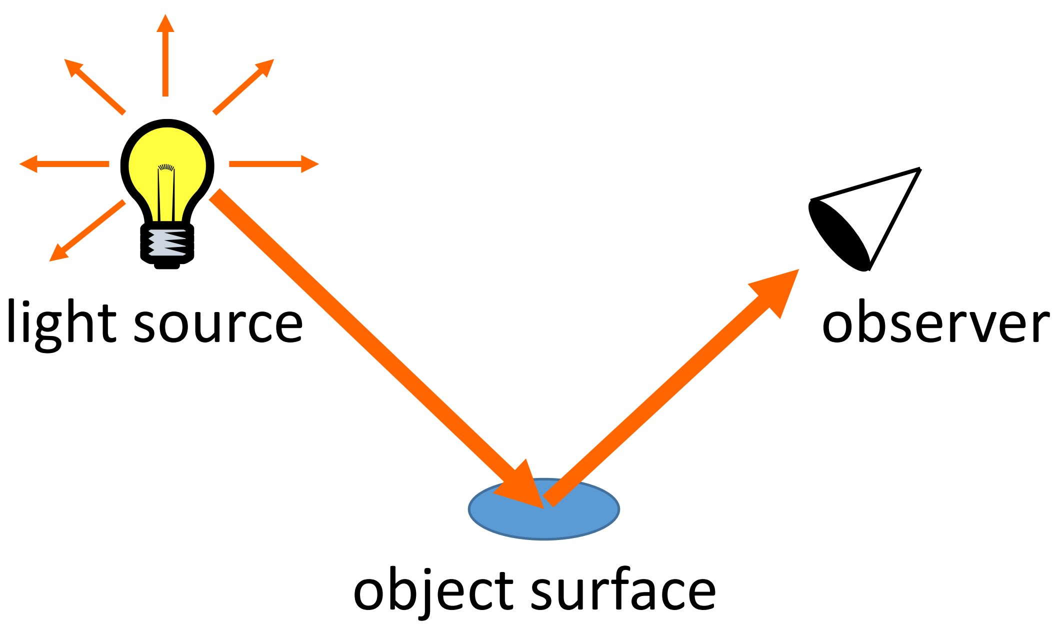
Physical quantity of light and light transport

Light energy transport

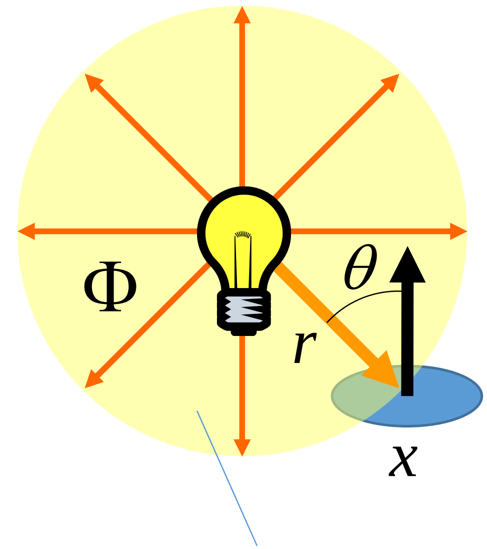
- In order to correctly treat "reflection" as a physical phenomenon,
  - Energy emitted from light source
  - Energy reaching object surface
  - Energy emitted from object surface

  should be considered.



Light energy on object surface

- Radiant flux(放射束) : Φ
  - Radiant energy per unit time
  - Unit : watt (W)
- Irradiance(放射照度) : $E(x)$
  - Light energy reaching object surface x
  - Radiant flux per unit area
  - Unit : W/m^2



Surface area : $4\pi r^2$

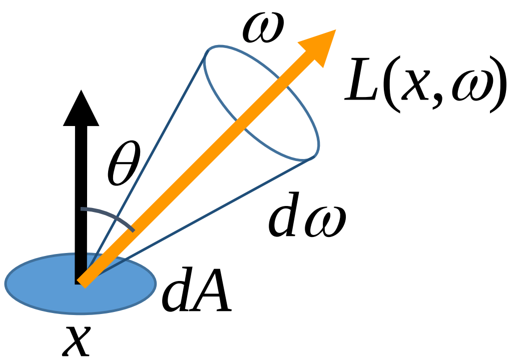
$$E(x) = \frac{\Phi \cos\theta}{4\pi r^2}$$

The received energy becomes smaller,
when the light source is far
and/or the surface tilts.

Emitted light energy

- Radiance(放射輝度) : $L(x,\omega)$
  - Light energy from x to ω direction
  - Radiance flux(放射束) per unit solid angle （立体角） and per unit area
  - Unit : W/m^{2}sr^2

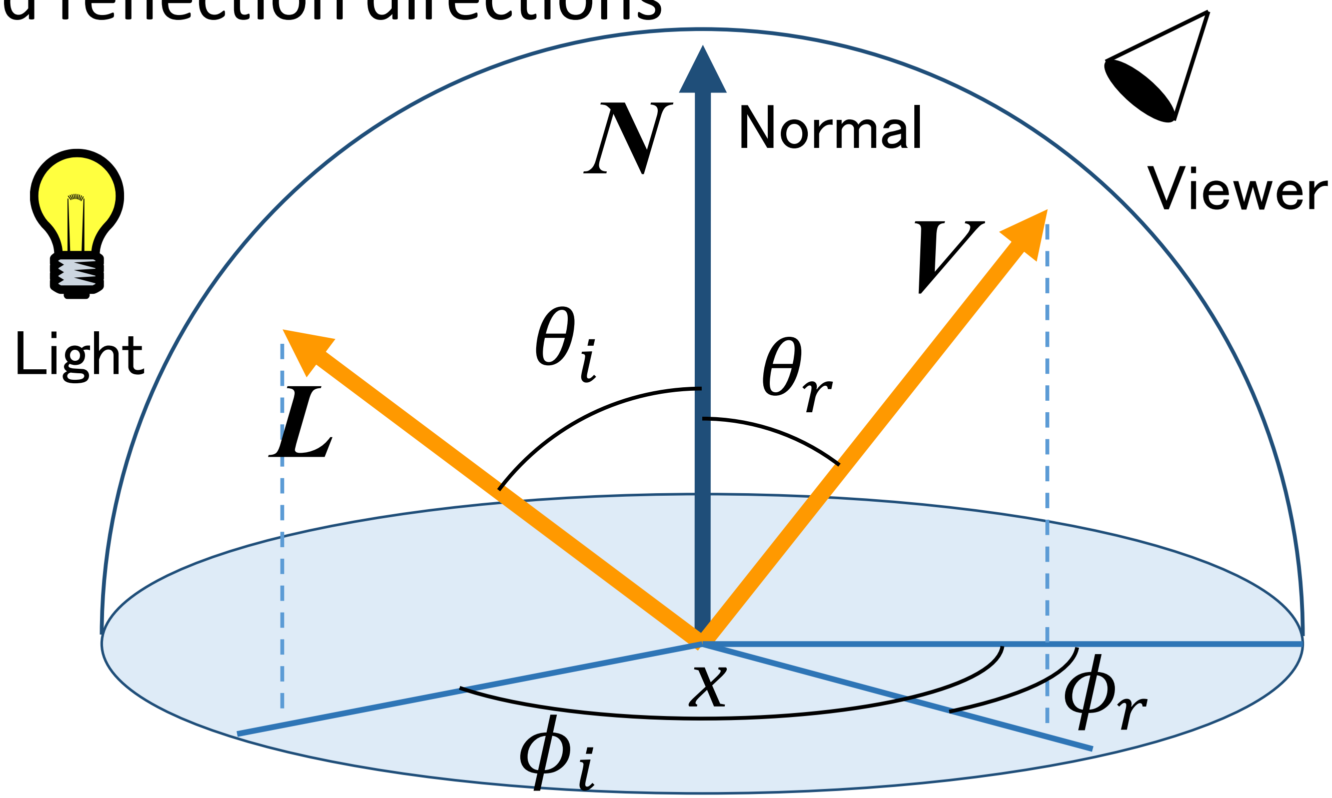
$$L(x,\omega) = \frac{d^2\Phi}{\cos\theta \, dA \, d\omega}$$



sr : steradian (unit of solid angle)

Modeling of reflection

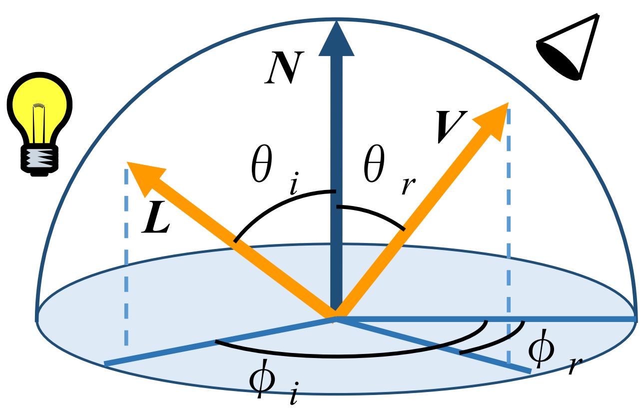
- How strongly does the light illuminated from the direction (θ_i, ϕ_i) at a certain point x reflects in the direction (θ_r, ϕ_r) ?
- Depends on **bidirectional**（双方向）of illumination and reflection directions



BRDF（双方向反射率分布関数）

- BRDF (**B**idirectional **R**eflection **D**istribution **F**unction)
- Ratio of radiance（出射光輝度）to irradiance（入射光照度）
- Usually, wavelength λ is omitted
  → In practice, defined by three color channels of RGB.

$$f_{BRDF}(x,\theta_i,\phi_i,\theta_r,\phi_r) = \frac{L_r(x,\theta_r,\phi_r)}{L_i(x,\theta_i,\phi_i)\cos\theta_i d\omega}$$



$$= \frac{L_r(x,\theta_r,\phi_r)}{E(x,\theta_i,\phi_i)d\omega}$$

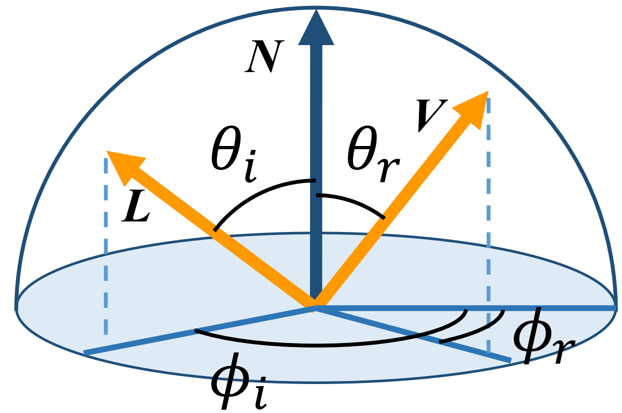
Angle parameters of BRDF

- Anisotropic reflection（異方性反射）
  - Four angle parameters

velvet

satin

brushed metal

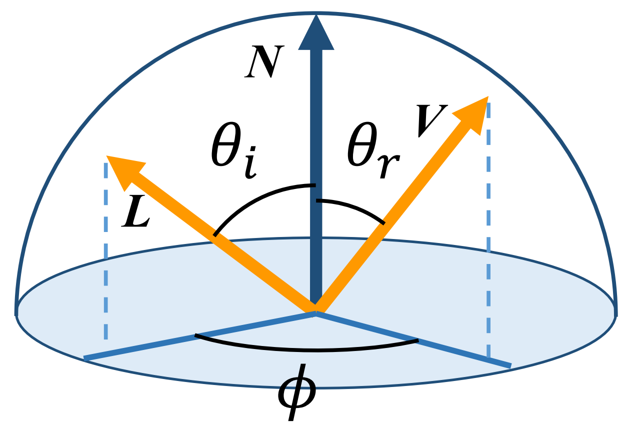


- Isotropic reflection（等方性反射）
  - Three angle parameters

$$f_{BRDF}(x, \theta_i, \phi_i, \theta_r, \phi_r)$$

⇓

$$f_{BRDF}^{isotropic}(x, \theta_i, \theta_r, \phi)$$

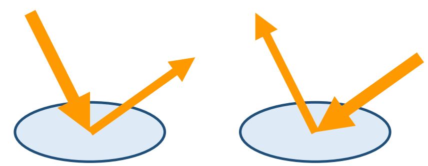


Conditions that BRDF should satisfy

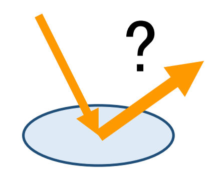
■Condition 1：Helmholtz reciprocity(相反性)

- Even if illumination direction and reflection direction are exchanged, the value does not change.
- Base for ray tracing

$$f_{BRDF}(x, L, V) = f_{BRDF}(x, V, L)$$



■Condition 2：Law of conservation of energy (エネルギー保存の法則)



- Do not emit energy more than entered.

$$\int_{\Omega^+} f_{BRDF}(x, L, V)(N \cdot L)dL \leq 1$$

Ω^+: Hemispherical surface seen from observation point

Calculation of radiance using BRDF

- Radiance(放射輝度) of reflected light at a point x on the object surface

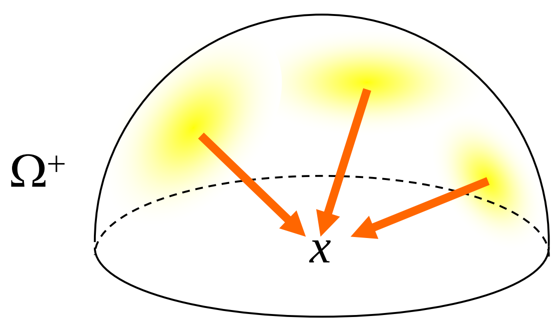
$$L_r(x,\theta_r,\phi_r) = \int_{\Omega^+} f_{BRDF}(x,\theta_i,\phi_i,\theta_r,\phi_r) L_i(x,\theta_i,\phi_i)\cos\theta_i d\omega$$

Radiance
(放射輝度)

Reflectance
(反射率)

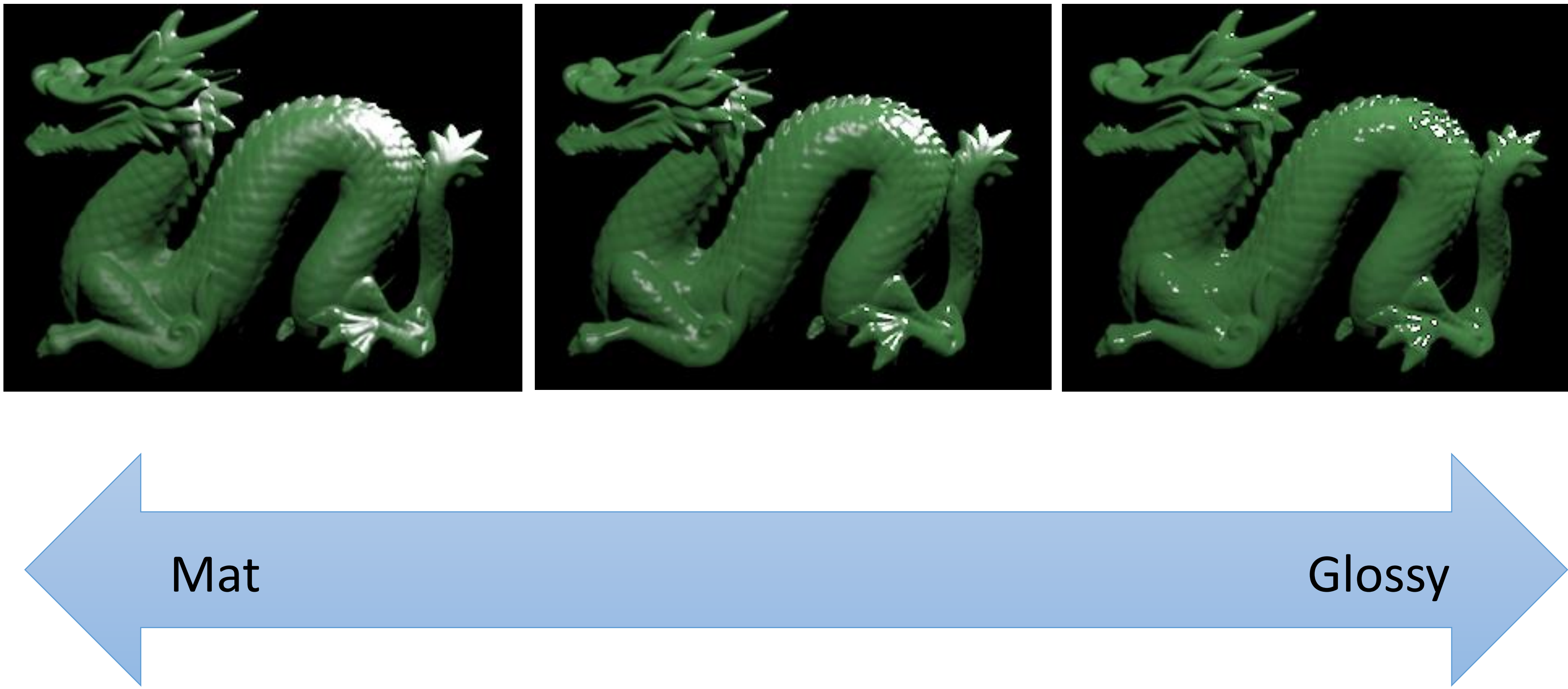
Irradiance
(放射照度)

Point x is illuminated from every direction on the hemisphere



Reflection Model

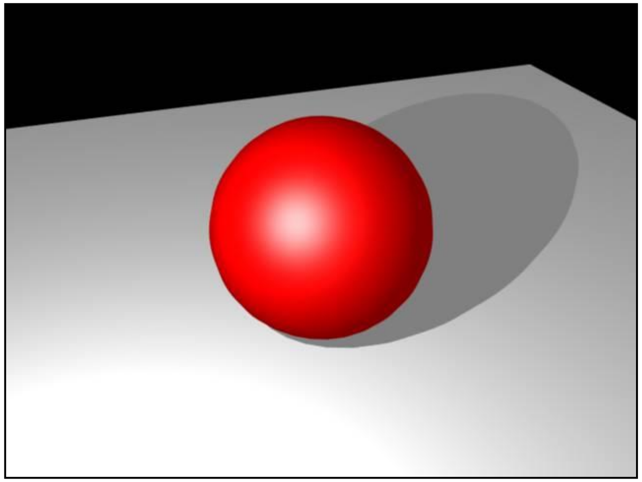
Difference in reflection properties

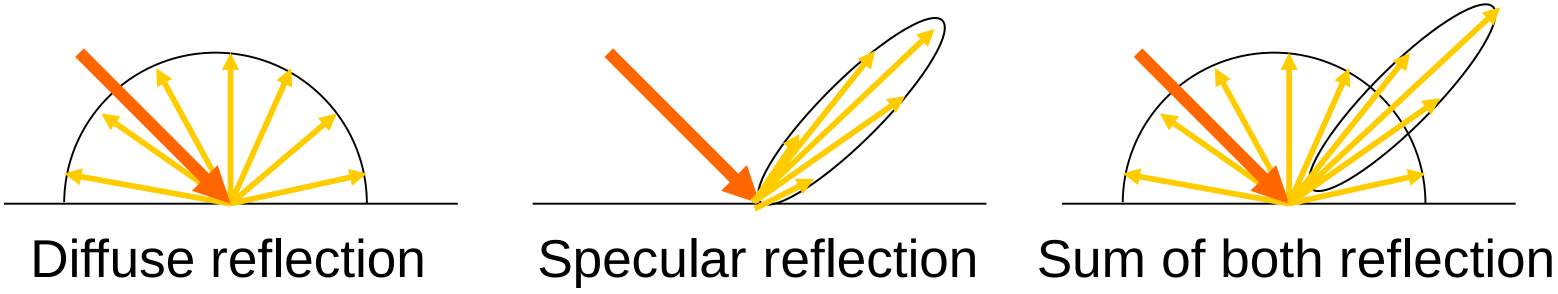


Dichromatic reflection model (Shafer 1985)

（2色性反射モデル）

- Reflected light = **Diffuse reflection** + **Specular reflection**
- Diffuse reflection（拡散反射）:
  - Reflection inside the surface layer
  - Object color
- Specular reflection（鏡面反射）：
  - Reflection at the border between air and surface layer
  - Light color





Model of diffuse reflection(拡散反射)

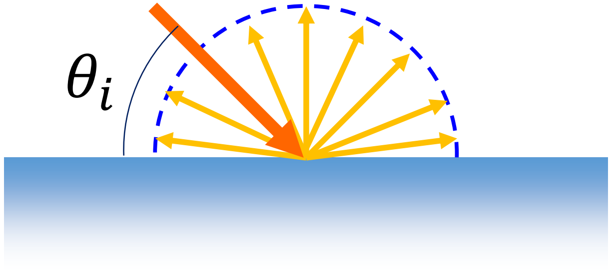
Johann Heinrich Lambert
(1728–1777)

Lambert model (1760)

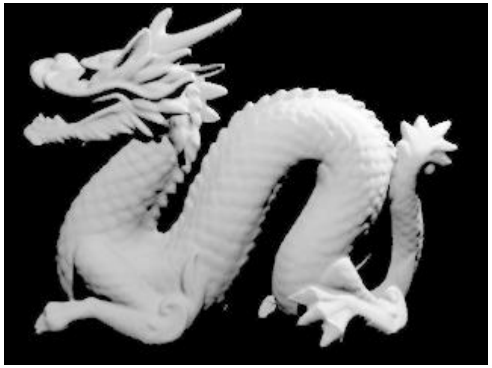
- Reflection with constant intensity in all directions
- Reflectance does not depend on illumination direction and observation direction

$$f_{BRDF}(\theta_i, \phi_i, \theta_r, \phi_r) = \rho_d$$

$$i = \rho_d \max(0, \cos\theta_i)$$

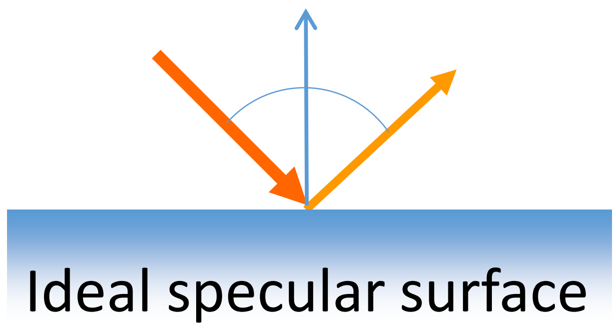


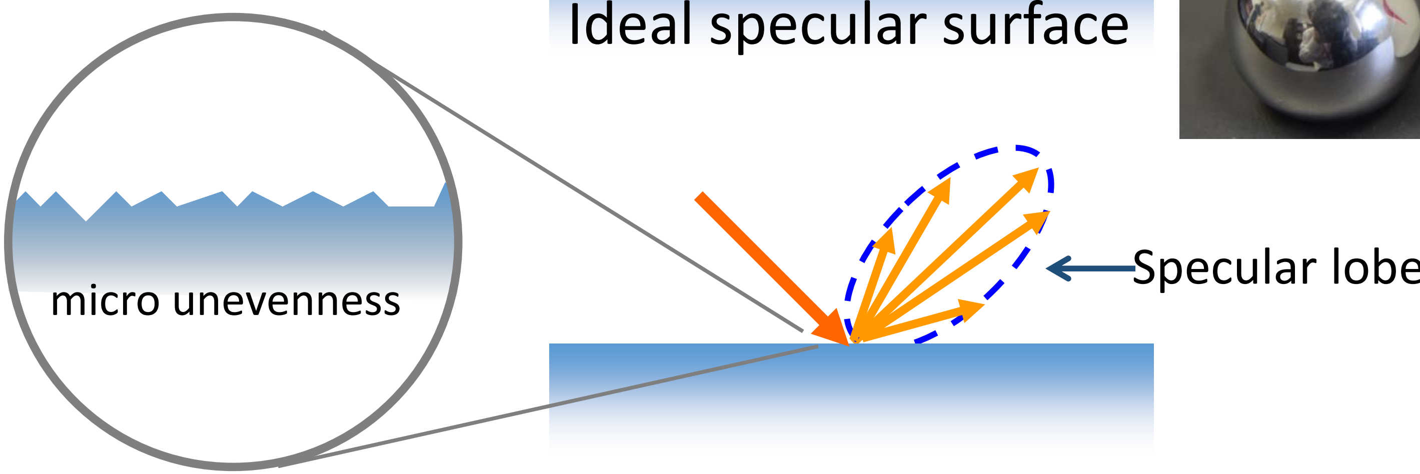
- ρ_d: Diffuse reflectance(拡散反射率)
- New models such as Oren-Nayar model (SIGGRAPH1994) have also been proposed, but still standard.



Specular Reflection(鏡面反射)

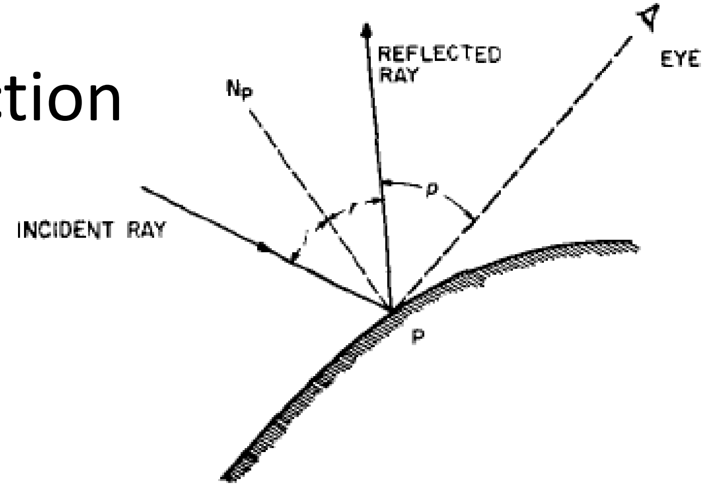
- Strongly observed in mirror direction(正反射方向)
- Due to micro unevenness on the surface, distribution becomes wider near the mirror direction.
- **Specular lobe(スペキュラーローブ)** is difficult to model accurately.

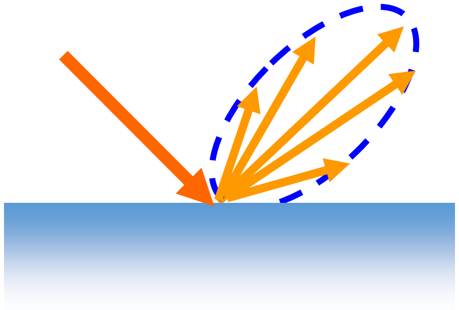


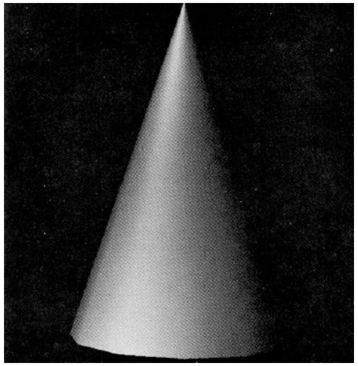


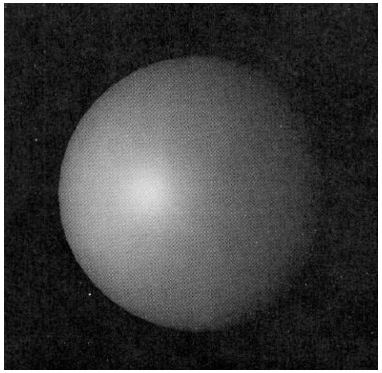
Phong Model

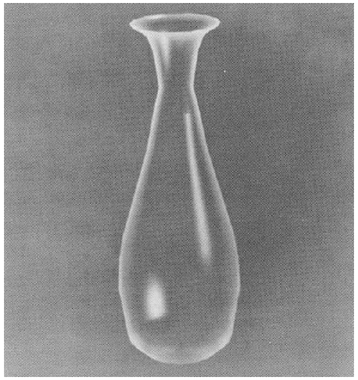
- Classical reflection model based on experience (SIGGRAPH1975)
  - It has a peak in the mirror direction
  - It weakens as angle moves away from mirror direction





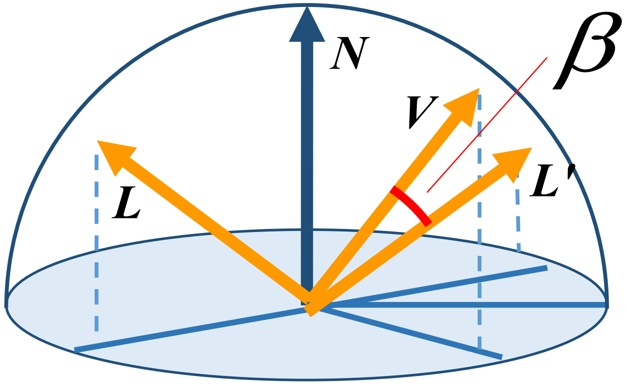






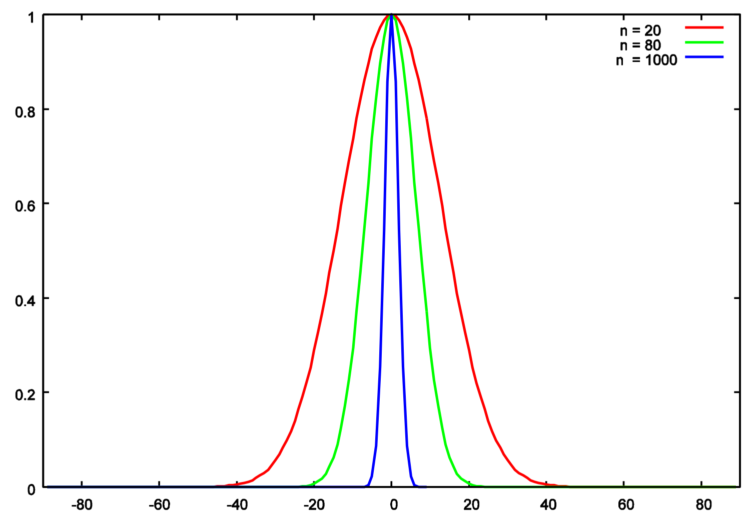
Phong Model

- Formulation by the power cosine of the angle(β) between the mirror direction($\boldsymbol{L}'$) of the light and the observation direction($\boldsymbol{V}$)



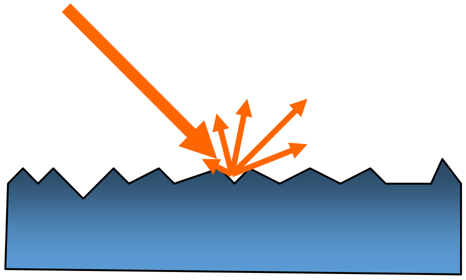
$$i = \rho_s \cos^n \beta$$

- ρ_s: Specular reflectance(鏡面反射率)
- n: Coefficient representing surface roughness
- Notice that it does not satisfy
  - Helmholtz reciprocity(相反性)
  - low of the conservation energy(エネルギー保存則)



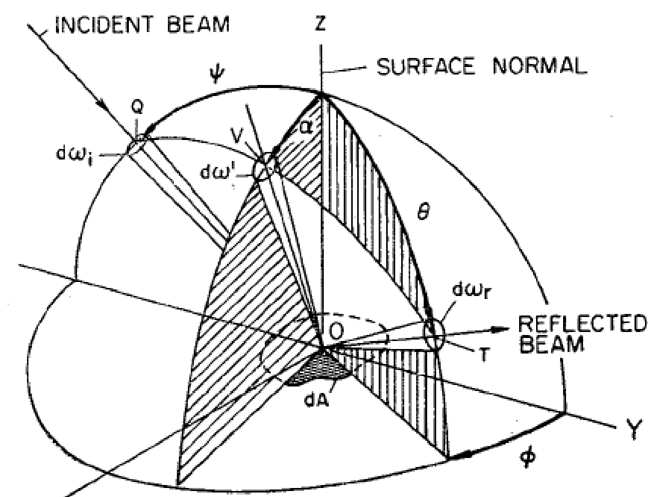
Model based on physical analysis

- Assume that object surface is a set of micro facets（微小面）
  1．How normal vector of micro facets varies?
  2．How surface point is occluded due to surface roughness?
  3．How Fresnel reflection(フレネル反射) effects?



Torrance-Sparrow Model

- A model based on the physical analysis which was developed earliest in the optical field (JOSA1967)
  - Modeling occlusion by micro facets and Fresnel reflection
- Represent off-specular(オフスペキュラー)
  - The peak of the specular reflection moves from the mirror direction
  - Title is "Theory for Off-Specular Reflection From Roughened Surfaces"



Formulation by Blinn (SIGGRAPH1977)

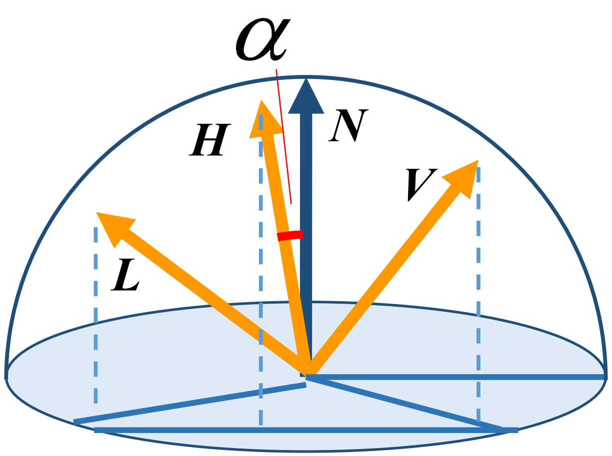
- Redefine Torrance-Sparrow model and apply to CG

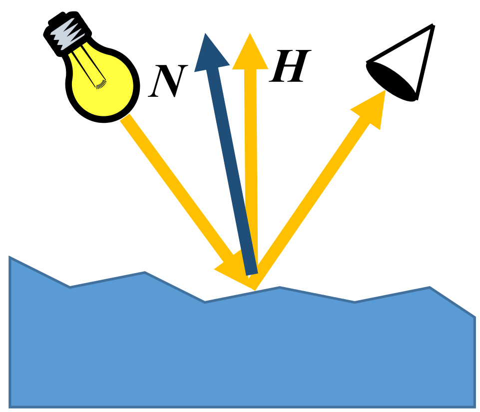
$$i = \rho_s \frac{DGF}{\mathbf{N} \cdot \mathbf{V}}$$

- D: Distribution function(法線分布)
  - Representing the variation of the surface normal
- G: Geometrical attenuation factor (幾何減衰)
  - Representing self-occulusion
- F: Fresnel reflection (フレネル反射)
  - Representing Fresnel reflections at boundary of different refractive indexes (屈折率)

D: Distribution Function(法線分布)

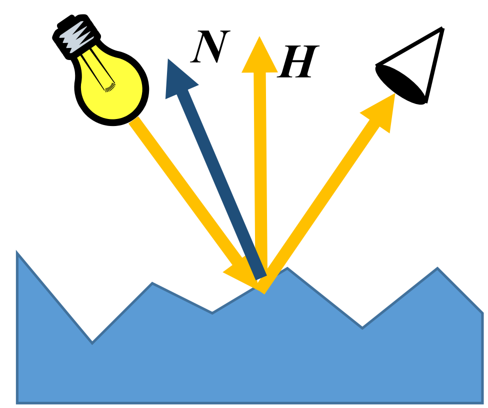
- A probability density function(確率密度関数) of an angle α formed by a half vector ($\boldsymbol{H}$) and a normal direction($\boldsymbol{N}$)
  - Half vector: bisector direction of the illumination and the observation directions
  - Assuming a set of micro facets that produce perfect specular reflection
  - How much do the normal vary to the half vector?





Smooth surface

$\boldsymbol{N}$ and $\boldsymbol{H}$ tend to coincide



Rough surface

$\boldsymbol{N}$ and $\boldsymbol{H}$ tend not to coincide

Various Distribution Functions (法線分布)

- Redefinition of the Phong model using half vector

$$D_1 = \cos^{n_1} \alpha$$

mini-report3: Why was β replaced by α?

- Gauss distribution used in Torrance-Sparrow model

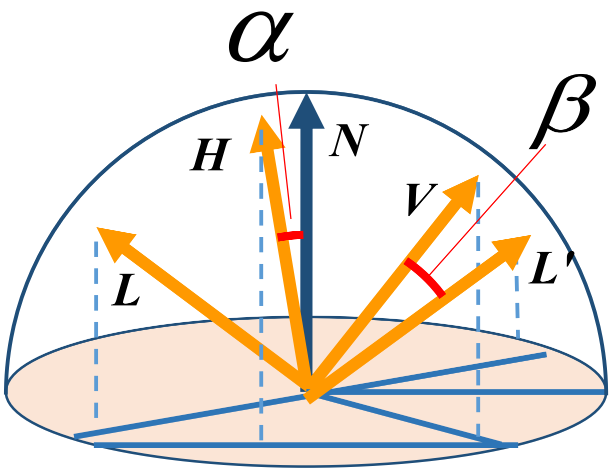
$$D_2 = e^{-(\alpha n_2)^2}$$

- Trowbridge-Reitz model

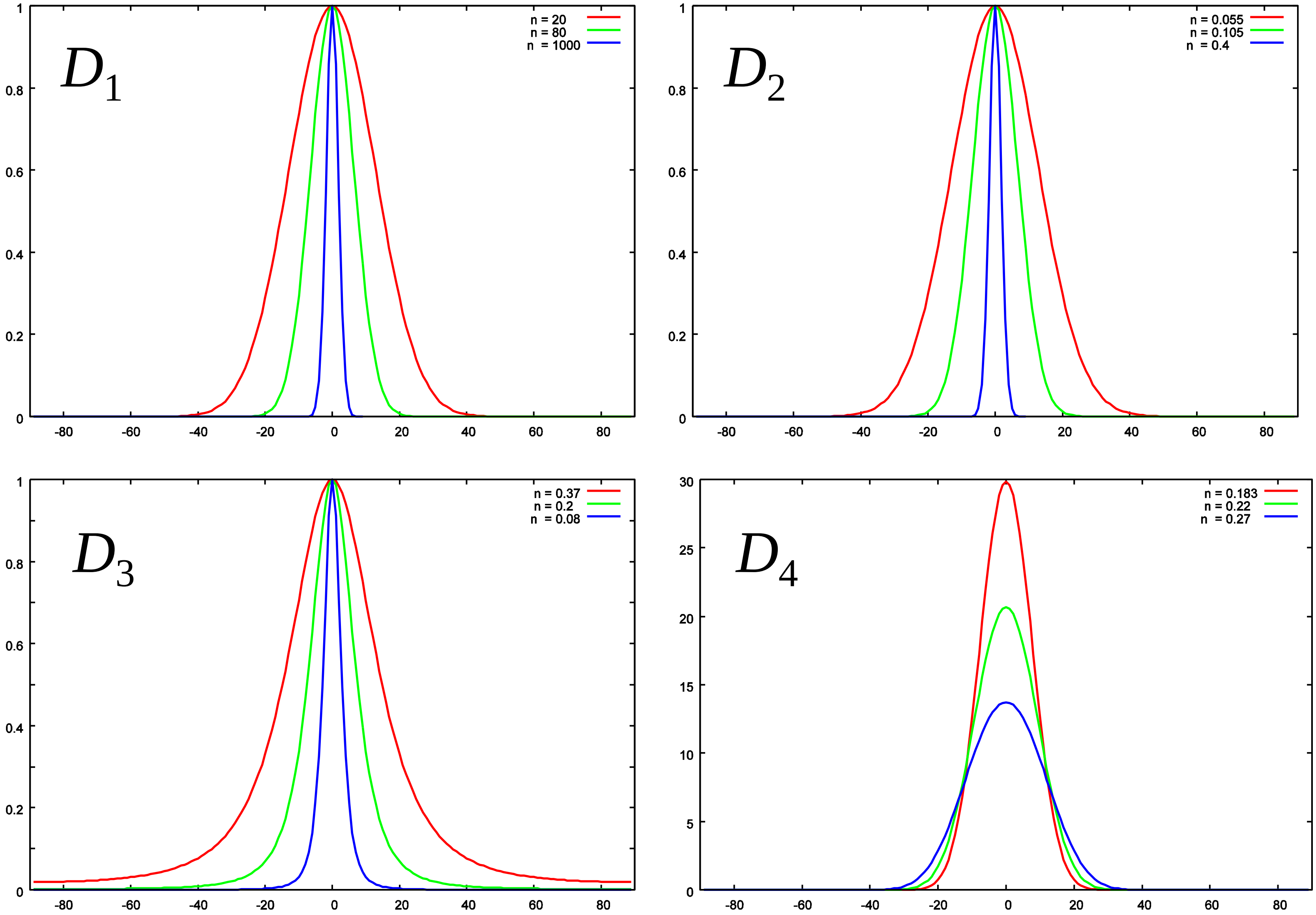
$$D_3 = \left(\frac{(n_3)^2}{\cos^2 \alpha ((n_3)^2 - 1) + 1} \right)^2$$

- Cook-Torrance model（Beckman distribution）

$$D_4 = \frac{1}{(n_4)^2 \cos^4 \alpha} e^{-\left(\frac{\tan^2 \alpha}{(n_4)^2} \right)}$$



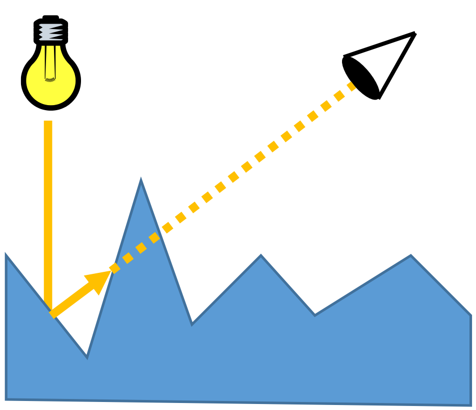
Examples of distribution function



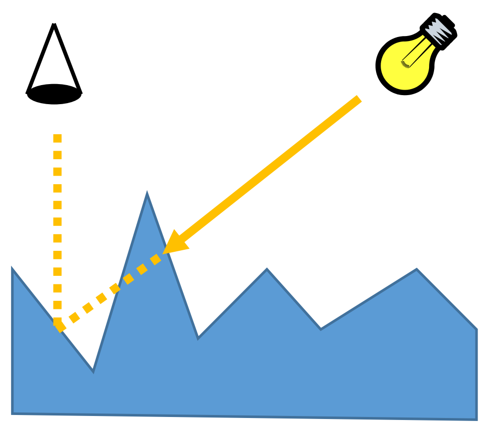
G: Geometrical Attenuation Factor (幾何減衰)

- Self-masking(自己遮蔽) and self-shadowing(自己陰影) caused by irregularities of micro facets
- As the illumination direction and/or observation direction approach tangent plane, attenuation increases

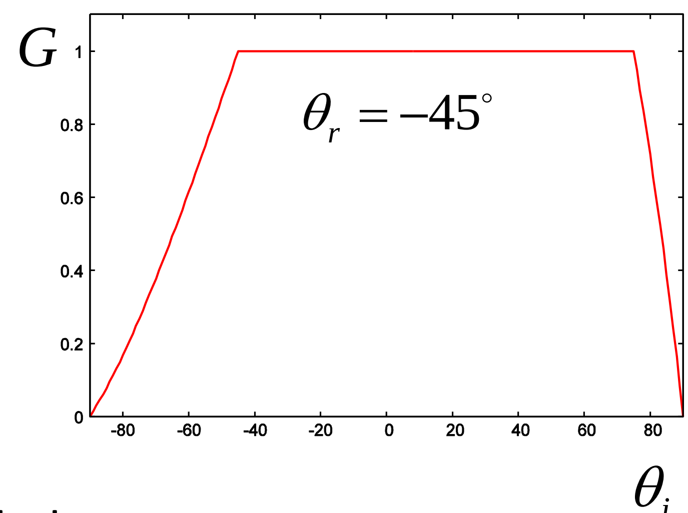
$$G = \min\left(1, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{V})}{\mathbf{V}\cdot\mathbf{H}}, \frac{2(\mathbf{N}\cdot\mathbf{H})(\mathbf{N}\cdot\mathbf{L})}{\mathbf{V}\cdot\mathbf{H}}\right)$$



Masking:
Reflected light is occluded

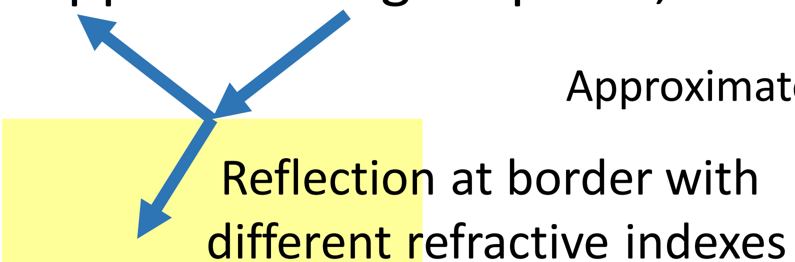


Shadowing:
Incident light is occluded

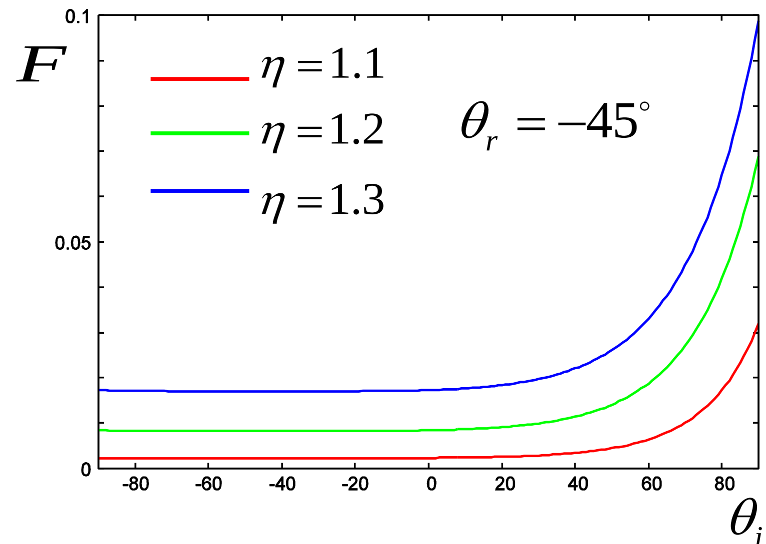


F: Fresnel Reflection(フレネル反射)

- Represent Fresnel reflection
  - Reflectance changes with refractive index(屈折率) and angle
  - As the illumination direction and/or observation direction approach tangent plane, reflectance becomes higher



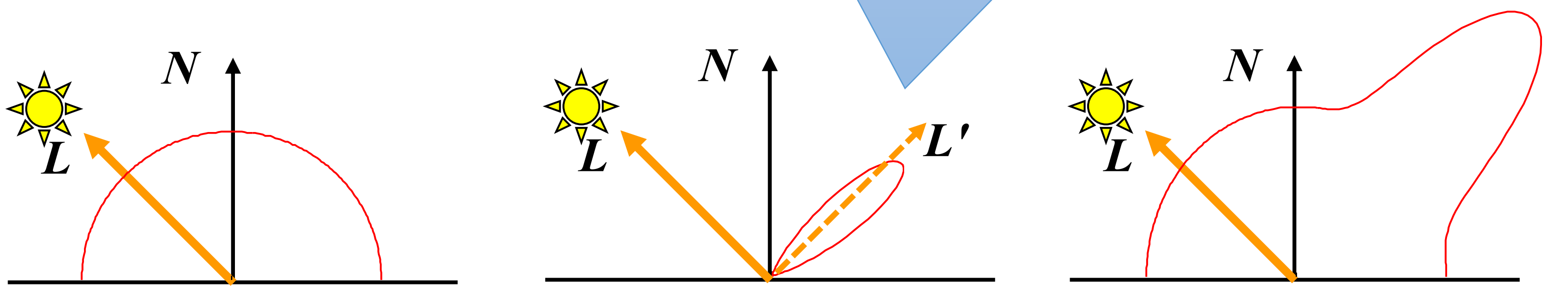
Approximate expression : $F = \frac{1}{2}\left\{\frac{\sin^2(\theta_i - \theta_r)}{\sin^2(\theta_i + \theta_r)} + \frac{\tan^2(\theta_i - \theta_r)}{\tan^2(\theta_i + \theta_r)}\right\}$



Example of Torrance-Sparrow Model

- When illumination direction θ_i=45

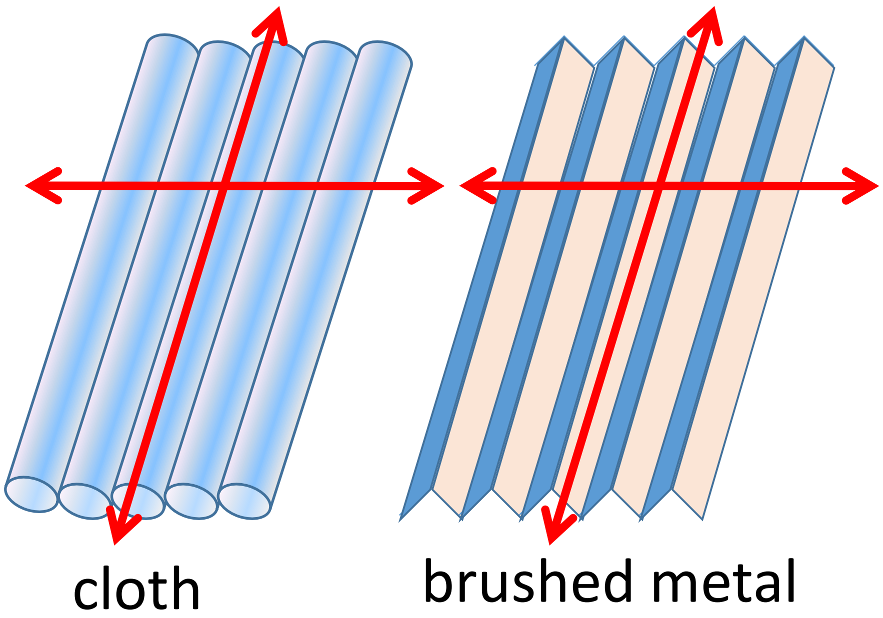
Off-specular
Peak at θ_r=47

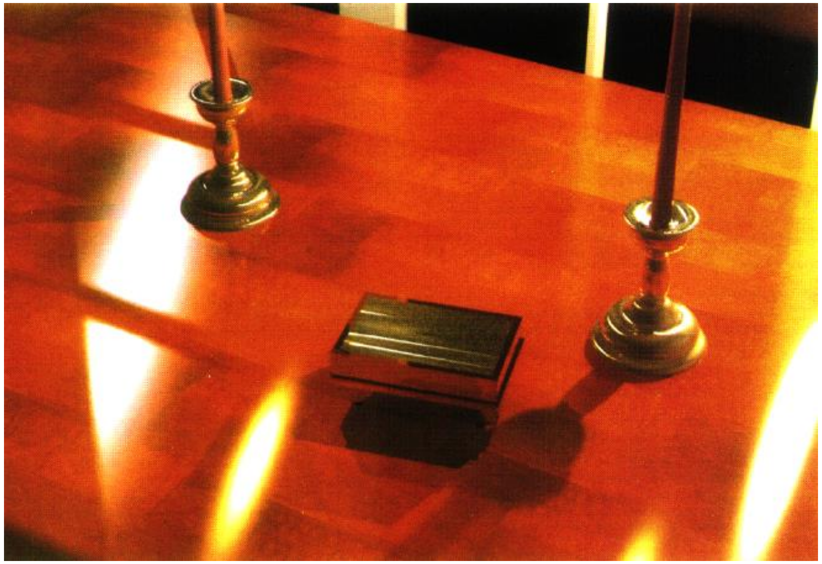


Diffuse reflection　Specular reflection　Sum of both reflection

Ward Model (SIGGRAPH1992)

- Representing anisotropic reflection(異方性反射)
  - Extension of distribution function in the Torrance-Sparrow model
  - Different roughness coefficients for parallel and vertical directions to the axis (fiber or brushing direction)



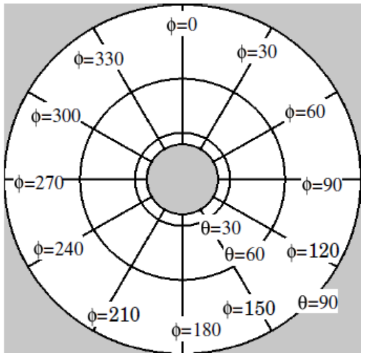


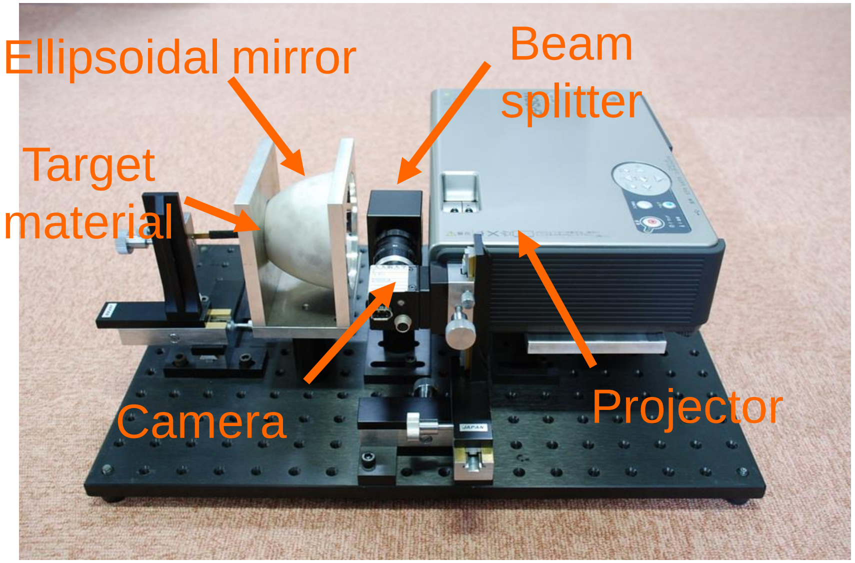
MERL BRDF Database

- Matusik et al., A Data-Driven Reflectance Model, ACM Transactions on Graphics (2003)
- Densely measured BRDFs of 100 different materials
  - plastic, metal, fabric, rubber, marble, ...
- Spherical target

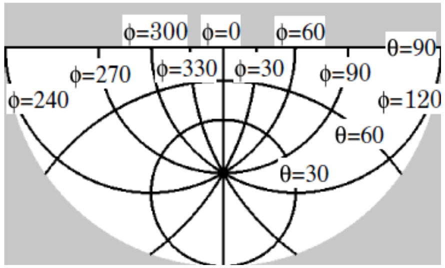
BRDF sampling devices

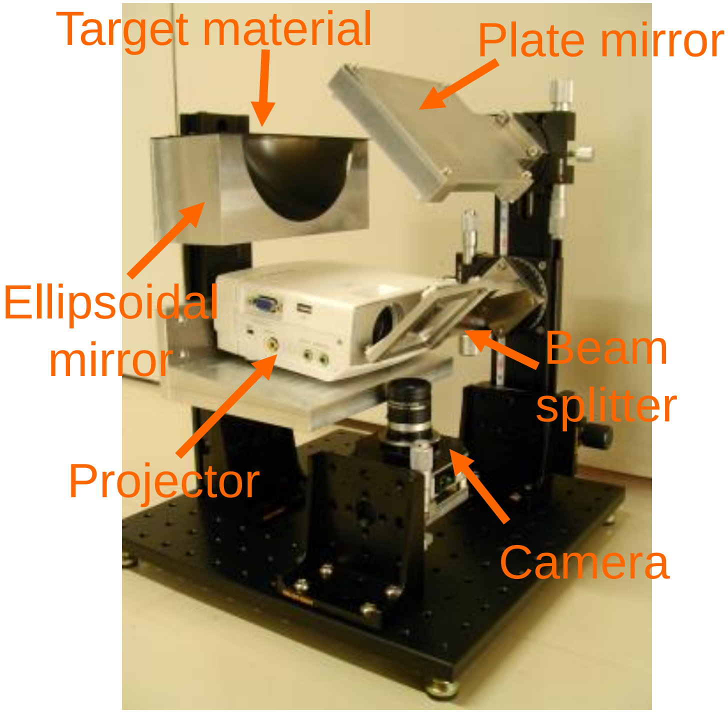
- Vertical setup (RCG-1)

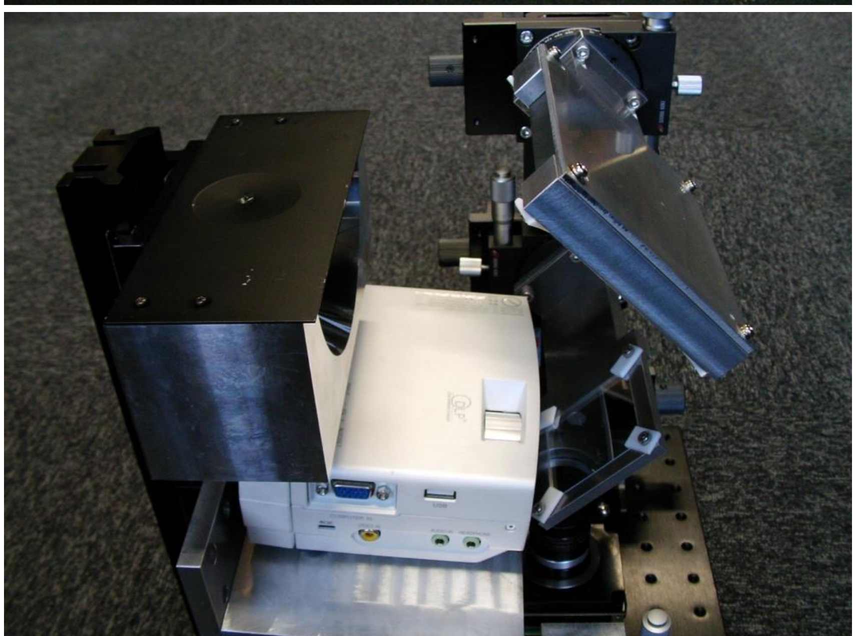


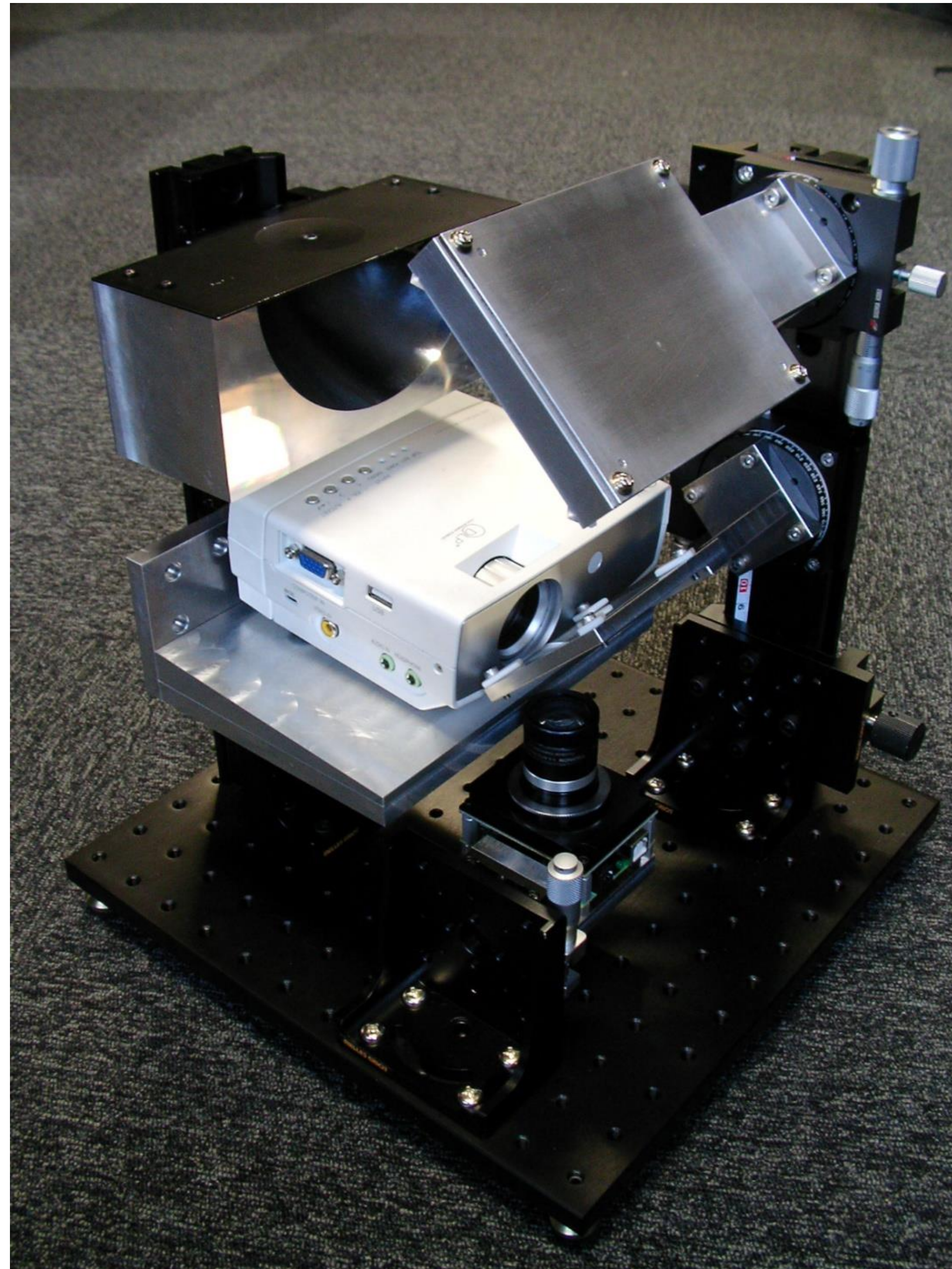


- Horizontal setup (RCG-2)



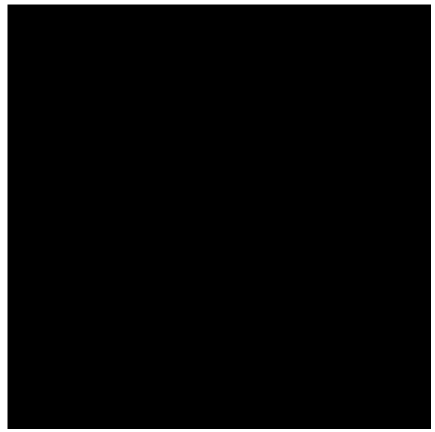






Sampled BRDF for CG

Real coin



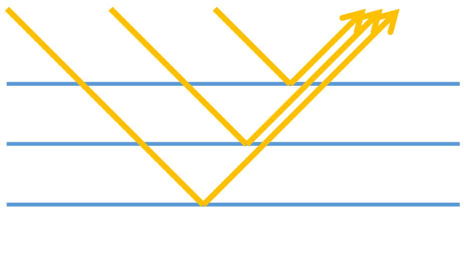
Sampled BRDF

Geometric shape

- isotropic reflection
- per one degree

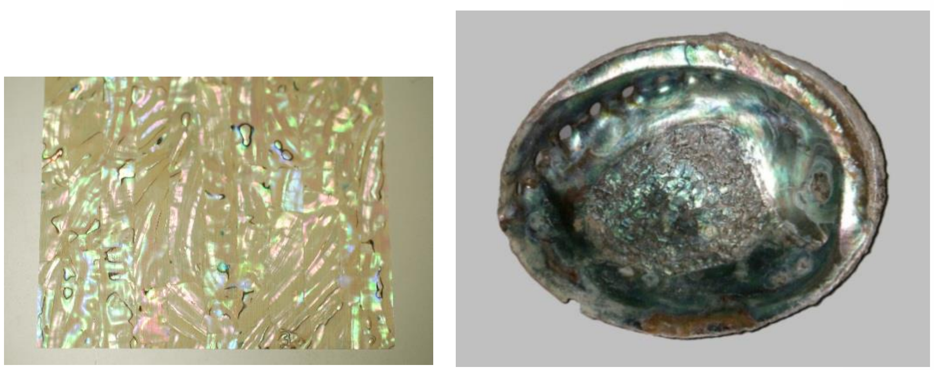
Structural color（構造色）

■Complex physical model



Multilayer interference
（多層膜干渉）

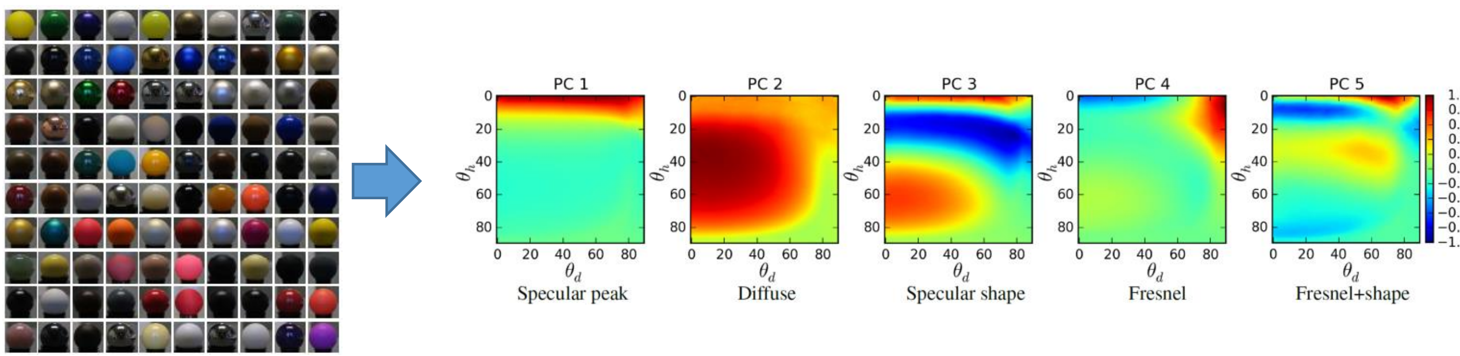
Structural color of nature



Mexican shell

Sparse sampling + PCA

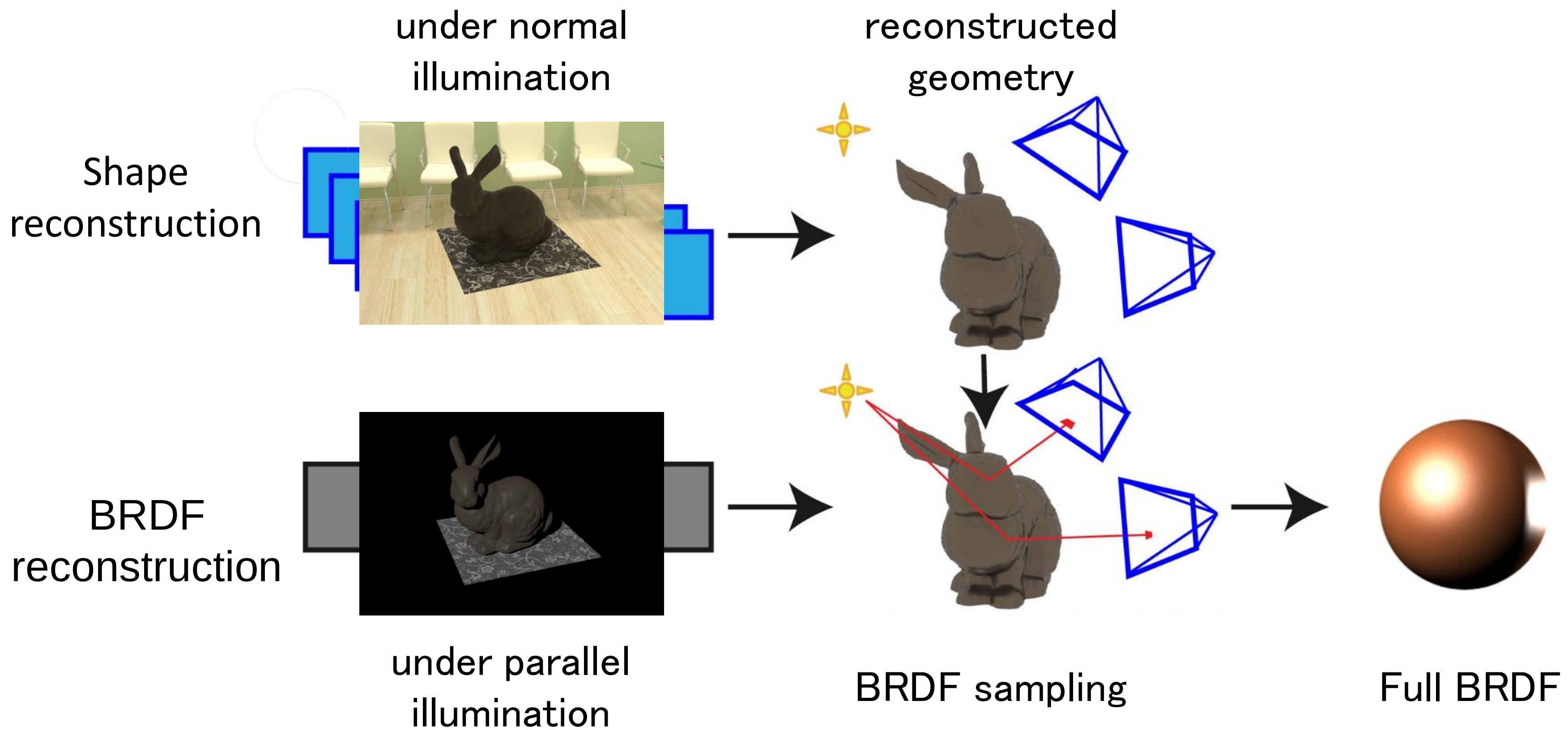
- PCA of MERL BRDF database
  - The BRDF of most objects can be represented by a linear sum of a small number of bases (BRDF is sparse)
  - BRSD measurement is equivalent to estimation of coefficients.



  - any BRDF= Σc_i base BRDF(i)

Nielsen et al., On Optimal, Minimal BRDF Sampling for Reflectance Acquisition, SIGGRAPH Asia 2015

BRDF sampling from real object



T. Ono, H. Kubo, T. Funatomi, Y. Mukaigawa, ``BRDF Reconstruction from Real Object using Reconstructed Geometry of Multi-view Images'', Proc. SIGGRAPH Asia2017.

T. Ono, H. Kubo, T. Funatomi, Y. Mukaigawa, ``BRDF Reconstruction from Real Object using Reconstructed Geometry of Multi-view Images'', Proc. SIGGRAPH Asia2017.

The Result of Simulated Experiment

Summary

- The early papers are still active.
  - diffuse reflection: 1760
  - specular reflection: 1967
- Recently, complete measurement of BRDF becomes possible.

Light Stage: University of Southern California

Final report

- Explain advantages and disadvantages to use complex and realistic BRDF model for CG and CV

CG	advantage	disadvantage
CV	advantage	disadvantage